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Regret theory: A new foundation

Enrico Diecidue *, Jeeva Somasundaram

Decision Sciences, INSEAD, Boulevard de Constance, 77305 Fontainebleau, France

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Abstract

We present a new behavioral foundation for regret theory. The central axiom of this foundation — trade-off consistency — renders regret theory observable at the individual level and makes our foundation consistent with a recently introduced empirical and quantitative measurement method. Our behavioral foundation allows deriving a continuous regret theory representation and separating utility from regret. The axioms in our behavioral foundation clarify that regret theory minimally deviates from expected utility by relaxing transitivity only.

JEL classification: D81

Keywords: Regret theory; Trade-off consistency; Expected utility; Transitivity

1. Introduction

Regret theory (Bell, 1982; Loomes and Sugden, 1982) is one of the most popular alternatives to expected utility (von Neumann and Morgenstern, 1947; Savage, 1954). Regret theory is based on the intuition that a decision maker (DM) choosing between two prospects (state-contingent payoffs), is concerned not only about the outcome he receives but also about the outcome he would have received had he chosen differently. When the outcome of the chosen prospect is less desirable than that of the foregone prospect, the DM experiences the negative emotion of regret.

* Corresponding author.
E-mail address: enrico.diecidue@insead.edu (E. Diecidue).

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The psychological content of regret in decision making has been extensively investigated[1] and evidence from neuroscience also supports the role of regret in decision making (Camille et al., 2004; Bourgeois-Gironde, 2010).

Bell (1982) and Loomes and Sugden (1982) formalized regret theory.[2] They postulate a continuous real-valued utility function u and a regret function Q to represent preferences over prospects. Consider prospects f and g with outcomes $f_1, \ldots, f_n$ and $g_1, \ldots, g_n$, under states $s_1, \ldots, s_n$, respectively, Loomes and Sugden (1982) represent preferences as follows:

$$f \succeq g \Leftrightarrow \sum_{i=1}^{n} p_i \cdot Q(u(f_i) - u(g_i)) \geq 0, \tag{1.1}$$

where p_i is the subjective probability of a state s_i. Eq. (1.1) is the focus of our paper. The utility function u captures the utility of outcomes. The Q function captures the attitude towards regret by transforming, state by state, the utility differences between the outcomes of the chosen (here f) and the foregone prospect (here g). A convex Q function amplifies large utility differences and prevents a DM from choosing a prospect with a possibility of large regret (or a large negative utility difference). Therefore, the psychological intuition of regret aversion is equivalent to a convex Q function. If the regret function Q is linear, the DM is not sensitive towards regret and Eq. (1.1) is equivalent to expected utility (EU).

A convex Q function allows regret theory to account for many of the empirical violations of EU e.g., the common consequence effect (Allais, 1953), preference reversals (Grether and Plott, 1979), and the coexistence of insurance and gambling (Kahneman and Tversky, 1979). Allowing for regret aversion also means that regret theory does not impose transitivity. Although transitivity is considered a central property of human behavior and is assumed in most normative theories of choice, including expected utility, Fishburn and LaValle (1988) argued that even thoughtful DMs exhibit intransitivities — of the kind predicted by regret theory — and that such intransitivities deserve serious consideration in normative theories of preference and choice.

Allowing for intransitivity entails a fundamental breakaway from any classical theory, requiring new insights into the concepts of maximization, indifference, and utility. Probably for this reason, for a long time no researcher attempted to obtain quantitative measurements of utility, and only a few axiomatizations existed but only of forms way more general than Eq. (1.1) and, hence, less tractable. Only some years ago, Bleichrodt et al. (2010) overcame these difficulties, and introduced the first quantitative measurements of the subjective parameters u and Q for Eq. (1.1).[3] Their evidence confirmed regret aversion (convex Q function) at the individual and aggregate levels. Only since then, concrete quantitative predictions based on regret theory could become conceivable. And only after that, based on their measurement method, we can now introduce the first preference foundation for regret theory (Eq. (1.1)).

Preference foundations for regret theory have been proposed by Fishburn (1989), Sugden (1993), and Quiggin (1994). Preference foundations allow for mathematically sound models and for the translation of observed preferences into the model's subjective parameters. For instance,

1 See, for example, Larrick (1993), Larrick and Boles (1995), Zeelenberg (1999), Connolly and Zeelenberg (2002), Connolly and Butler (2006) and Zeelenberg and Pieters (2007).

2 Fishburn (1982, 1984) presented a mathematical theory of skew-symmetric bilinear (SSB) preferences that is equivalent to a general form of regret theory.

3 This availability of the measurement technique allowed Baillon et al. (2015) to experimentally explore intransitivities under regret theory.

the original prospect theory proposed by Kahneman and Tversky (1979), never received a preference foundation and, as discovered later, was not theoretically sound. Also, the maxmin EU proposed by Hurwicz (1951), and discussed by Luce and Raiffa (1957), became popular only after Gilboa and Schmeidler (1989) behavioral foundation.

The existing behavioral foundations in the domain of regret (Fishburn, 1989; Sugden, 1993; Quiggin, 1994) axiomatize, however, only a related general version of regret theory (Loomes and Sugden, 1987), where the utility and regret function are not separated. It is behaviorally important to separate the utility and regret function, conceptually and for the purpose of tractability, because regret applications in economics and management science rely on utility differences (see Subsection 1.1 and especially our application in Section 4). For example, policy makers require utilities debiased from regret for the purpose of prescriptive decisions. Existing decision models such as cumulative prospect theory (Tversky and Kahneman, 1992, CPT) and the disappointment aversion model (Gul, 1991, DA), for example, become operational only when the utility function can be disentangled from the probability distortion (under CPT) and disappointment aversion (under DA). Similarly, to make regret theory operational — for example to do comparative statics with regret function — it is necessary to separate utility and the regret function. The existing foundations (Fishburn, 1989; Sugden, 1993; Quiggin, 1994) also do not allow for a continuous representation, which is necessary for economic modeling, quantitative measurements, and applications. The axioms of the existing foundations are also complex and prevent intuitive comparison with other theories such as expected utility. For example, Sugden (1993) relaxes transitivity from the Savage axioms, and adds a complex P_4^*, which lacks a clear empirical meaning. Therefore, an intuitive preference foundation for Eq. (1.1), which is the original, most tractable and almost exclusively used version of regret theory is called for.

This paper presents (Section 3) the first behavioral foundation of Eq. (1.1) with continuous utility and regret functions. The key axiom in the foundation is trade-off consistency, a natural generalization of de Finetti's book making principle (de Finetti, 1931; Wakker, 2010, §4.14). The trade-off consistency axiom is also the basis for the trade-off method, a measurement technique used to measure the subjective parameters of expected utility (Wakker and Deneffe, 1996), cumulative prospect theory (Abdellaoui, 2000; Bleichrodt and Pinto, 2000) and, more importantly, regret theory (Bleichrodt et al., 2010). An appealing feature of modern preference foundations is their close relationship with the measurement of the subjective parameters of the theory, amounting to consistency requirements for the theory (Wakker, 2010). The resulting foundations, unlike Savage (1954), do not need an infinite state space and do not require a presupposed notion of probability (as in Anscombe and Aumann, 1963). These foundations do require a "rich" outcome set but the monetary domain, available in economic applications, readily satisfies this requirement. By providing a behavioral foundation based on trade-off consistency, this paper offers a theoretical justification not only for regret theory but also for the measurement technique of Bleichrodt et al. (2010).

Our behavioral foundation derives regret theory by weakening the transitivity axiom of expected utility (Köbberling and Wakker, 2003) into a new *dominance-transitivity (d-transitivity)* axiom. The d-transitivity axiom imposes transitivity only when there is a dominating (or dominated) prospect in a set of three prospects. By restricting transitivity this way, the behavioral foundation shows exactly that regret theory deviates minimally from expected utility. Our behavioral foundation also clarifies the conclusions of Bikhchandani and Segal (2011) by showing that transitive regret coincides with expected utility. Indeed, the proof of the behavioral foundation relies on showing the interrelationship between trade-off consistency and Fishburn's (1990) independence axiom. Fishburn's (1990) independence axiom is a complex but powerful axiom, which by itself (in the presence of few technical axioms) implies a state-dependent regret representation.[4]

1.1. Applications

The simple structure and intuitive appeal of Eq. (1.1) make it suitable for applications. For example, Bell (1983) applies the representation to define risk premiums for decision analysis. Somasundaram and Diecidue (2017) build on this definition to analyze the role of feedback in affecting risk attitudes. Loomes and Sugden (1983) use Eq. (1.1) to rationalize preference reversals. They show that intransitivities induced by regret aversion can explain preference reversals, a result empirically validated by Loomes et al. (1991). Assuming a separate utility u and regret function Q in the representation, allowed comparative statics with the regret function. For example, Braun and Muermann (2004) apply regret theory, as in Eq. (1.1), to understand the demand for insurance and show that anticipated regret can explain the frequently observed preference for low deductibles. Muermann and Volkman (2007) and Michenaud and Solnik (2008) apply regret theory to explain the disposition effect and the role of regret-risk in currency hedging. Eq. (1.1) has also been applied to consumer behavior (Nasiry and Popescu, 2012) and newsvendor problem (Schweitzer and Cachon, 2000).

Eq. (1.1) has been modified to suit the specific context of other applications. For example, Filiz-Ozbay and Ozbay (2007) and Engelbrecht-Wiggans and Katok (2008) simplify the original representation of Eq. (1.1) and consider only the regret of negative utility differences to explain overbidding in first price auctions. A modified version of Eq. (1.1) has been applied to health decisions (Smith, 1996), finance (Gollier and Salanié, 2006), newsvendor (Perakis and Roels, 2008) and consumer behavior (Diecidue et al., 2012). Recently, regret theory has also been extended theoretically: Sarver (2008) extends regret theory to preferences over menus, Maccheroni et al. (2012) extend regret to social decisions and interdependent preferences, and Gollier (2015) characterizes the concept of regret-risk aversion. The case for the historical importance of Eq. (1.1) is made by Bleichrodt and Wakker (2015).

2. Preliminaries

Consider a finite *state space* $S = \{s_1, \ldots, s_n\}$. The *outcome set* is $\mathbb{R}$, with real numbers designating amounts of money. Prospects are state-contingent outcomes mapping the state space S to $\mathbb{R}$. *Prospects* are denoted by lower case letters $(f, g, \ldots)$ and outcomes are usually denoted by Greek letters α, β, γ and δ with and without subscripts, or by Roman letters with subscripts such as x_1. A prospect $f = (f_1, \ldots, f_n)$ is identified with the function assigning outcome f_j to each state j. Let $\mathbb{R}^n$ denote the set of all prospects. Consider a *preference relation* $\succeq$ over the set of prospects. *Strict preference* $\succ$, *indifference* (or equivalence) $\sim$, and *reverse preferences* $\preceq$ and $\prec$ are defined as usual. The *valuation function* V represents preference $\succeq$ on the set of prospects. That is, for every prospect f, g, $f \succeq g$ if and only if $V(f) \geq V(g)$. For an outcome α, a prospect f, and a state s_i, $\alpha_i f$ denotes the prospect f with f_i replaced by α. Prospects with probabilities specified are indicated by $\alpha_{p_i} f$, where p_i is the probability of state s_i.

We introduce standard conditions for the preference relation $\succeq$. In particular, $\succeq$ is *complete* if $f \succeq g$ or $g \succeq f$ for all $f, g \in \mathbb{R}^n$, and $\succeq$ is *transitive* if $f \succeq g$ and $g \succeq h$ implies $f \succeq h$, for

[4] The Fishburn independence axiom is stronger than the related axioms such as the von Neumann–Morgenstern independence (von Neumann and Morgenstern, 1947) and the sure thing principle (Savage, 1954).

all $f, g, h \in \mathbb{R}^n$. A state s_i is *null* if $\alpha_i f \sim \beta_i f$ for all prospects f and outcomes α and β, and *non-null* otherwise. *Weak monotonicity* holds if for all prospects f and $g \in \mathbb{R}^n$, if $f_i \geq g_i$ for all $i = 1, ..., n$, then $f \succeq g$. *Strong monotonicity* holds if, for all prospects f and $g \in \mathbb{R}^n$, $f \succ g$ whenever $f_i \geq g_i$ for all i and $f_i > g_i$ for a state s_i that is non-null on $\mathbb{R}^n$. The preference relation $\succeq$ is *continuous* if the sets $\{f \in \mathbb{R}^n : f \succeq g\}$ and $\{f \in \mathbb{R}^n : f \preceq g\}$ are closed subsets of $\mathbb{R}^n$ for each $g \in \mathbb{R}^n$.

We now provide a formal definition of regret theory as in Eq. (1.1). Loomes and Sugden (1982) defined regret theory as follows:

Definition 1. *Regret theory* holds if there exist a continuous strictly increasing utility function $u : \mathbb{R} \to \mathbb{R}$, subjective probabilities p_i, and a continuous strictly increasing skew-symmetric regret function $Q : \mathbb{R} \to \mathbb{R}$ such that Eq. (1.1) holds.

Skew symmetry of Q means $Q(\alpha) = -Q(-\alpha)$ for all α. The convexity (resp., concavity) of the Q-function indicates *regret aversion* (resp., *regret seeking*). Loomes and Sugden (1987) analyzed a more general model of Eq. (1.1): They postulate a continuous function $\psi : \mathbb{R}^2 \to \mathbb{R}$ such that

$$f \succeq g \Leftrightarrow \sum_{i=1}^{n} p_i \cdot \psi(f_i, g_i) \geq 0, \tag{2.1}$$

where f, g are prospects in $\mathbb{R}^n$ and p_i is the subjective probability of state s_i. The function ψ is unique up to scale — that is, it can be replaced by any other function $\psi' = a\psi$, $a > 0$ without affecting preferences — and it satisfies the following two restrictions:

1. The function ψ is strictly increasing (resp., strictly decreasing) in its first (resp., second) argument: for any outcome γ, if $\alpha > \beta$ then $\psi(\alpha, \gamma) > \psi(\beta, \gamma)$ and $\psi(\gamma, \alpha) < \psi(\gamma, \beta)$.
2. The function ψ is skew symmetric: for all α and β, $\psi(\alpha, \beta) = -\psi(\beta, \alpha)$. Skew symmetry implies, for all outcomes α, $\psi(\alpha, \alpha) = 0$.

Fishburn (1989) and Sugden (1993) provided axiomatic foundations for Eq. (2.1) using Savage's approach, i.e., using axioms which require an infinite state space and no atoms in the subjective probability distribution. However, the representations they derived did not give a continuous ψ.

Fishburn (1990) provided an axiomatic foundation for a state dependent version of Eq. (2.1). This representation postulates continuous functions $\psi_i : \mathbb{R}^2 \to \mathbb{R}$ such that

$$f \succeq g \Leftrightarrow \sum_{i=1}^{n} \psi_i(f_i, g_i) \geq 0, \tag{2.2}$$

where again f, g are prospects in $\mathbb{R}^n$. The functions ψ_i are continuous and satisfy restrictions 1 and 2. Note that the function ψ_i depends on the state, and therefore the representation in Eq. (2.2) is the state dependent version of Eq. (2.1). The representations in Eq. (2.1) and Eq. (2.2) play a key role in the proof of our behavioral foundation for Eq. (1.1), the most popular and tractable version of regret theory.

The $\sim_t$ relation described next (Köbberling and Wakker, 2003) is central to this paper. It can be interpreted as a strength of preference relation over the outcomes revealed by ordinal preferences over acts, as we now explain. For outcomes $\alpha, \beta, \gamma, \delta$ we write

$$\alpha\beta \sim_t \gamma\delta \quad \text{or} \quad \alpha \ominus \beta \sim_t \gamma \ominus \delta \tag{2.3}$$

if there exist prospects f, g and a non-null state s_j, such that

$$\alpha_j f \sim \beta_j g \quad \text{and} \quad \gamma_j f \sim \delta_j g. \tag{2.4}$$

In Eq. (2.3), $\alpha \ominus \beta$ is interpreted as receiving outcome α instead of outcome β. The interpretation of $\sim_t$ is that receiving α instead of β is an equally good improvement as receiving γ instead of δ. That is, both of these improvements exactly offset the receipt of f instead of g on states other than state s_j.

Before we define trade-off consistency and preference trade-off consistency, we now introduce the $\succeq_t$ and $\succ_t$ notation:

$$\alpha\beta \succeq_t \gamma\delta \tag{2.5}$$

if there exist prospects f, g and a non-null state s_j, such that

$$\alpha_j f \succeq \beta_j g \quad \text{and} \quad \gamma_j f \preceq \delta_j g. \tag{2.6}$$

Similarly, $\alpha\beta \succ_t \gamma\delta$, if at least one of the preferences $\alpha_j f \succeq \beta_j g$ or $\gamma_j f \preceq \delta_j g$ is strict. Consider prospects f, g, x, and $y \in \mathbb{R}^n$, the trade-off consistency is defined as follows.

Definition 2. Trade-off (TO) consistency holds on $\mathbb{R}^n$ if $\alpha_i f \sim \beta_i g$, $\gamma_i f \sim \delta_i g$ and $\alpha_j x \sim \beta_j y$ imply $\gamma_j x \sim \delta_j y$.

TO consistency implies that the trade-off (say, $\alpha\beta \sim_t \gamma\delta$) between outcomes holds irrespective of corresponding prospects and states, provided the required indifference exists. Trade-off consistency's intuition appeals to rational ways of making decisions: by weighing the pros and cons of a prospect, state by state. It discourages noncompensatory heuristics such as just going for certainty. In this sense, the intuition of trade-off consistency is similar in spirit to state-wise comparisons used in regret theory. Therefore, using trade-off consistency to axiomatize regret theory captures the decision process that underlies regret theory. The behavioral implications of trade-off consistency are extensively discussed by Köbberling and Wakker (2004) and Wakker (2010, pp. 111, 138). Trade-off consistency was developed by Wakker (1984) as a generalization of de Finetti's book making principle (de Finetti, 1931). Trade-off consistency is a natural generalization of de Finetti's additivity when the linearity of utility scale is no longer available.

The TO consistency in Definition 2 is a stronger version of the condition used before (Wakker, 2010). Köbberling and Wakker (2003) refer to Definition 2 as strong indifference trade-off consistency, which — along with completeness, monotonicity, and continuity — has been used in the literature to axiomatize expected utility (EU) and non-EU theories (Köbberling and Wakker, 2003; Alon and Schmeidler, 2014). The trade-off consistency based foundations, as opposed to Savage (1954), require only a finite state space but require a "rich" outcome set (connected topological space, see Fishburn, 1970 and Krantz et al., 1971). As opposed to Anscombe and Aumann (1963), in the trade-off consistency based approach, richness of the outcome set does not necessarily derive from mixture operations on a space of lotteries. Therefore the notion of probability is not presupposed, and no restrictions are imposed on the decision maker's behavior under risk. Comparing trade-off consistency to other standard axioms in the literature (Savage, 1954; Anscombe and Aumann, 1963), we remark that trade-off consistency is stronger than the sure-thing principle or the von Neumann–Morgenstern independence axiom. Another important feature of the trade-off consistency axiom is its closeness with the measurement technique — the trade-off method. The trade-off consistency axiom ensures that the trade-off method can be used consistently for measurement. Bleichrodt et al. (2010) used the trade-off method to measure the subjective parameters of regret theory. Therefore, because the measurement technique (trade-off method) coincides with the axiom (trade-off consistency), the theory's measurement acts as an experimental test of the theory.

Definition 3. *Preference trade-off consistency* holds on $\mathbb{R}^n$, if no four preferences exist of the form $\alpha_i f \succeq \beta_i g$, $\gamma_i f \preceq \delta_i g$, $\alpha_j x \preceq \beta_j y$ and $\gamma_j x \succ \delta_j y$.

The preference trade-off consistency was introduced by Wakker (1984) primarily for the purpose of normative defenses of expected utility. A condition similar to preference trade-off consistency was used in early papers (Krantz et al., 1971 and Tversky, 1977). Note that preference trade-off consistency can also be defined using the $\succeq_t$ notation. Preference trade-off consistency holds on $\mathbb{R}^n$, if there are no four outcomes α, β, γ and δ such that, $\alpha\beta \succeq_t \gamma\delta$ and $\gamma\delta \succ_t \alpha\beta$.

Definition 4. Prospect f *strictly dominates* prospect g, if $f_i \geq g_i$, for all i and if $f_i > g_i$ for a state s_i that is non-null. We denote strict dominance by $\succeq_{SD}$.

By strong monotonicity, $f \succeq_{SD} g$ implies $f \succ g$. The $\succeq_{SD}$ notation in Definition 4 will allow to define a new transitivity axiom called *dominance-transitivity* abbreviated as *d-transitivity*.

Definition 5. *D-transitivity* holds for all prospects $f, g,$ and $h \in \mathbb{R}^n$, if $[f \succeq_{SD} g$ and $g \succeq h]$ implies $f \succ h$ and if $[f \succeq g$ and $g \succeq_{SD} h]$ implies $f \succ h$.

D-transitivity states that if prospect f is preferred to prospect g, then increasing the outcome(s) of prospect f or decreasing the outcome(s) of prospect g should not affect the preference. In other words, suppose the DM prefers a particular prospect, then improving the DM's preferred prospect or worsening the less preferred prospect should not change preference. This axiom makes normative sense under regret theory: improving (resp., worsening) the DM's preferred (resp., less preferred) prospect should not induce any regret that could alter the DM's choices. Descriptively, when there is one dominating (resp., dominated) prospect among three prospects, the DM would be able to order the three prospects easily. The d-transitivity axiom has a prior appearance in the literature: Stoye (2011) refers to d-transitivity as "transitive extension of monotonicity" and applies it to menu dependent preferences.

To conclude the preliminaries, we briefly refer to the Bleichrodt et al. (2010) measurement, i.e., the first quantitative measurement of Eq. (1.1). This two-stage measurement allows one to separate and measure utility (in the first stage) and regret function (in the second stage) directly from choice, without imposing any parametric form. Appendix A describes the two stages in detail. Although Bleichrodt et al. (2010) empirically demonstrated the possibility of measuring u and Q, their method lacks a theoretical justification. In the next section, we introduce a behavioral foundation based on trade-off consistency: the axiom rationalizes Bleichrodt et al.'s use of trade-off method for consistent measurement.

3. A new foundation for regret theory

We now state the representation theorem for regret theory.

Theorem 1. *The following two statements are equivalent:*

1. *Regret theory holds with a strictly increasing continuous utility function u, subjective probabilities p_i, and a strictly increasing, skew symmetric continuous regret function Q.*
2. *$\succeq$ satisfies:*
   (i) *completeness;*
   (ii) *d-transitivity;*
   (iii) *strong monotonicity;*
   (iv) *continuity; and*
   (v) *trade-off consistency.*

Furthermore, the subjective probabilities are uniquely determined, the utility function u is unique up to unit and level, and the regret function Q is unique up to unit.

Outline of the proof. The proof of this theorem is in Appendix D. We next discuss the steps of the proof in details because that clarifies its constructive nature, consistent with the empirical measurement. In a nutshell: The claim that statement 1 implies statement 2 is straightforward (Lemma 1). To prove that statement 2 implies statement 1, we first derive a state-dependent regret representation. Then we derive regret theory from the state-dependent representation.

***Step 1.* Deriving a state-dependent regret representation**

A state-dependent regret representation was derived in Fishburn (1990) in Eq. (2.2) (see Appendix C for detailed illustration) using the following axioms: (a) structure (connected and separated topological space) and non-triviality, (b) continuity, (c) independence, and (d) non-extremality. To derive a state-dependent regret representation, we have to show that our axioms (i) to (v) imply Fishburn's (1990) axioms. Note that axioms (a), (b), and (d) of Fishburn (1990) are technical. Because our outcome set is $\mathbb{R}$, our axioms (iii) and (iv) (continuity and strong monotonicity) imply continuity and non-extremality axiom of Fishburn (1990). Our outcome set $\mathbb{R}$ is also endowed with the necessary structure that the first part of axiom (a) in Fishburn (1990) requires. The second part of axiom (a) (non-triviality) is implied by strong monotonicity. Hence, the only task that remains for deriving a state-dependent regret, is to show that our axioms (i) to (v) imply the independence axiom of Fishburn (1990). This independence axiom is a complex but powerful axiom: In the presence of other technical axioms, it guarantees a state-dependent regret representation. Because independence axiom of Fishburn (1990) is close to preference trade-off consistency (discussed in Axiom (c), Appendix C), our next step will be to derive preference trade-off consistency from axioms (i) to (v).

a. Deriving preference trade-off consistency

To derive preference trade-off consistency from axioms (i) to (v), we use Proposition 30 of Köbberling and Wakker (2003). It shows that if weak order, strong monotonicity, continuity and trade-off consistency hold, then preference trade-off consistency holds. As we do not have weak order, we cannot use the results of Köbberling and Wakker (2003) directly. Hence, in Proposition 1 we use d-transitivity, a weaker version of transitivity (that regret theory satisfies), along with our other axioms ((i), (iii), (iv) and (v)) to derive preference trade-off consistency.

b. Deriving Fishburn's (1990) independence axiom

Fishburn's (1990) independence axiom is close to preference trade-off consistency. When prospects have common gauge outcomes,[5] Fishburn's (1990) axiom is equivalent to preference trade-off consistency for a given state. Therefore, preference trade-off consistency implies Fish-

[5] Two pair of prospects have common gauge outcomes if the outcomes of each prospect, from a particular pair, precisely matches the outcomes of a prospect from the other pair, across all states, except the one state considered. For instance, the

burn's (1990) independence for prospects with a common gauge. However, if the prospects do not have a common gauge, we cannot derive Fishburn independence axiom directly from preference trade-off consistency. This concerns the complicated part of the proof. In Lemma 4, we show that if Fishburn's (1990) independence axiom is violated, then preference trade-off consistency is violated, even though the prospects may not have a common gauge. The d-transitivity, strong monotonicity, and continuity axioms play an important role in this part of the proof. In Lemma 4 we show that in the presence of axioms (i) to (v), preference trade-off consistency implies the Fishburn (1990) independence axiom.

Having derived the Fishburn (1990) independence axiom, Lemma 5 shows that our axioms (i) to (v) imply the axioms of Fishburn (1990). We thereby derive a state-dependent regret representation, i.e., for prospects $f = (f_1, \dots, f_n)$ and $g = (g_1, \dots, g_n)$, $f \succeq g$ if and only if $\sum \psi_i(f_i, g_i) \geq 0$, where ψ_i is continuous and skew-symmetric. Because $\succeq$ satisfies strong monotonicity, ψ_i is strictly increasing in the first argument and strictly decreasing in the second argument (see Lemma 5).[6]

***Step 2.* Deriving regret theory from a state-dependent regret representation**

To derive regret theory from the state-dependent regret representation of Fishburn (1990), we first derive the subjective probabilities and then we derive the utility and regret function. The steps are discussed in detail below.

a. Deriving subjective probabilities and a skew symmetric ψ function

Trade-off consistency is the key axiom that allows deriving the subjective probabilities p_i and the ψ function. Because trade-off consistency imposes specific indifferences across states, it allows showing that the different ψ_i functions are related by an affine function (see Lemma 6 and 7). The proportionality between the different ψ_i functions allows deriving the unique subjective probabilities p_i.

b. Deriving the regret function Q and utility function u from the ψ function

Trade-off consistency allows building standard sequences of outcomes. The standard sequence of outcomes are outcomes equally spaced in ψ units. Such a sequence is elicited by fixing the state and gauge outcomes of the prospects. For instance, by fixing an outcome α_0, an outcome α_1 is elicited such that $\alpha_{1_i} f \sim \alpha_{0_i} g$ for a state s_i, and prospects f, g. After α_1 is elicited, we find α_2 such that $\alpha_{2_i} f \sim \alpha_{1_i} g$. Repeated application of this procedure allows us to build standard sequence of outcomes $\alpha_0, \dots, \alpha_k$. Our proof mainly relies on showing that any two standard sequence outcomes separated by same subscript j — such as α_i, α_{i+j} and α_k, α_{k-j} — are equidistant in ψ units. This allows deriving the utility function u and the regret function Q from the skew-symmetric ψ function (See Lemma 8). Since ψ_i is unique up to unit in Fishburn (1990) representation, ψ is also unique up to unit, i.e., ψ can be replaced by a function $a \cdot \psi$ with a

prospects with the preferences $\alpha_i f \succeq \beta_i g$, $\gamma_i f \succeq \delta_i g$ have common gauge outcomes f_j and g_j across all states except one state. However, prospects with the preferences, $\alpha_i f \succeq \beta_i g$, $\gamma_i f' \succeq \delta_i g'$ do not have common gauge outcomes – the outcomes in the states $j \neq i$ of a prospect $\alpha_i f$ (resp., $\beta_i j$) are not the same as outcomes in the states $s_j \neq s_i$ of $\gamma_i f'$ (resp., $\delta_i g'$). The preference (resp., indifference) TO consistency can be applied only to the former case.

[6] Fishburn's (1990) independence axiom (in the presence of technical axioms (a), (b), and (d)) imply a state-dependent regret representation even without the completeness of the preference relation $\succeq$. His result contributes to the literature exploring the relationship between completeness and continuity (Schmeidler, 1971; Dubra, 2011, and Karni, 2011). Specifically, Fishburn (1990) is the only paper that shows that completeness implies continuity even when $\succeq$ is not a preorder. We tried to drop the completeness axiom from our set of axioms, however, completeness of $\succeq$ is needed to derive Fishburn's (1990) independence from our other axioms, and Fishburn's (1990) independence or transitivity of $\succeq$ are also required to derive completeness. Since we assume neither Fishburn's (1990) independence nor transitivity, we require completeness as a separate axiom.

real $a > 0$, without affecting the preference. As $\psi(f_i, g_i) = Q(u(f_i) - u(g_i))$, Q is also unique up to unit. Because the regret theory representation depends only on the difference between the utilities of the outcomes, the function u is unique up to both unit and level, i.e., it can be replaced by a function $u' = a \cdot u + c$ (with real $a > 0$, c) without affecting preference.

Thus, we have proved the claim that part 2 implies part 1. Note that the trade-off consistency axiom plays the key role in our proof. First, trade-off consistency helps deriving preference trade-off consistency, which in turn enables deriving the state-dependent regret representation. Then, since trade-off consistency imposes trade-off indifferences across states, it enables deriving the subjective probabilities. Finally, the standard sequence of outcomes built using trade-off consistency helps deriving the utility function u and the regret function Q.

To sum up: Our behavioral foundation for Eq. (1.1) allows deriving a continuous regret representation. In addition, it allows for a separation of utility from the regret function, i.e., we are able to disentangle u and Q functions from the skew-symmetric ψ function for the first time. Because policy makers are primarily interested in measuring utility, our foundation — along with the measurement method (Bleichrodt et al., 2010) — allows for direct utility measurement independent of regret effects.

Fishburn (1989) weakened the transitivity axiom of Savage (1954) to derive a non-continuous regret theory representation without the possibility of separating utility and regret functions. Sugden (1993) built on Fishburn (1989) and, as a consequence, the regret theory representation derived is not continuous and does not allow for the separation of utility and regret functions.[7] Quiggin (1994) proposed a regret theory representation based on irrelevance of state-wise dominated alternative (ISDA) axiom,[8] but the representation is essentially identical to Fishburn (1989) and Sugden (1993). Our behavioral foundation is the first to derive a regret theory representation as in Eq. (1.1), with a subjective probability p_i, a strictly increasing and continuous utility function u, and a strictly increasing, skew-symmetric, and continuous regret function Q. Note also that the proof of our behavioral foundation mainly relies on the strength of trade-off consistency axiom. Since trade-off consistency is stronger than the vNM independence (or the sure thing principle), we cannot derive regret theory by using d-transitivity instead of transitivity in the Anscombe-Aumann or Savage set-up.

Alon and Schmeidler (2014) emphasize simplicity and transparency as a necessary condition for normative and descriptive interpretations of an axiom and they also introduce a coarse measure of opaqueness to measure the simplicity of an axiom. In this respect, the axioms of our behavioral foundation are simple, transparent, and tractable compared to the existing foundations for regret theory.[9] According to Alon and Schmeidler (2014), simplicity and transparency have normative and descriptive advantages. Normatively, they allow a DM to construct his unknown preferences and justify regret theory as a valid procedure for decisions. Descriptively, our simple

7 Hayashi (2008) and Sarver (2008) also provided a behavioral foundation for regret theory, but they consider a model different from that of Loomes and Sugden (1982). Recently, Tserenjigmid (2015) built a behavioral foundation for the intra-dimensional comparison (IDC) heuristic (Tversky, 1969). Although the IDC heuristic is different from regret theory conceptually, the IDC representation is related to the state-dependent regret representation of Eq. (2.2).

8 The ISDA axiom — similar to *d-transitivity* — states that the addition (or removal) of a dominated prospect into a choice set should not affect the preference. For instance, if f is most preferred among f, g and $g \succeq_{SD} h$, then $f \succeq h$. This is identical to d-transitivity. However, unlike the d-transitivity, the ISDA does not imply – if $f \succeq_{SD}$ and $g \succeq h$, then $f \succ h$.

9 For instance the $P4^*$ axiom in Sugden (1993) requires more than 15 variables, 15 preferences, and two implications. Such an axiom is not only difficult to understand but also difficult to test experimentally.

axioms can be tested experimentally and can be used to convey that regret may be more prevalent than it may appear at first sight.

Our behavioral foundation also provides the theoretical foundation for the Bleichrodt et al. (2010) measurement. *Trade-off consistency* guarantees the requirement for the first stage — the utility function measurement. For the second stage of the measurement, *continuity* guarantees that there exists a Q function. Strong monotonicity and d-transitivity ensure that the Q function is unique. Thus, the axioms of our behavioral foundation — trade-off consistency, d-transitivity, strong monotonicity, continuity, and completeness — are sufficient to guarantee the consistency of the trade-off based measurement in Bleichrodt et al. (2010) method.

Comparing the representation theorem of EU (Köbberling and Wakker, 2003) with the representation theorem of regret theory (Theorem 1), the only difference is that *transitivity* is replaced by *d-transitivity* axiom. Because d-transitivity allows for intransitivity in a DM's preference, the representation theorem for regret theory is a more general case of EU. Bikhchandani and Segal (2011) showed that transitive regret theory coincides with EU and it is not consistent with non-EU theories of transitive choices. Our representation theorem agrees with the conclusions of Bikhchandani and Segal (2011): When imposing transitivity to our set of axioms we get expected utility. To provide more intuition, we also illustrate graphically the relationship between EU and regret theory in Appendix E. The appendix also discusses the ramifications of dropping transitivity altogether (when a theory is "completely" intransitive), i.e., when the d-transitivity axiom is dropped from our behavioral foundation. Thus, by restricting transitivity to a subset of $\mathbb{R}^n$, d-transitivity naturally separates regret theory from EU. We next discuss an example to illustrate how regret theory satisfies d-transitivity but violates full transitivity.

Example. Consider the following parametric specification for regret theory: $n = 3$, $Q(\alpha) = \alpha^2$ if $\alpha \geq 0$ and $Q(\alpha) = -\alpha^2$ for $\alpha < 0$, $u(\alpha) = \alpha$ and $p(s_i) = 1/3$. Consider the prospects $f = (25, 25, 15)$, $g = (15, 30, 20)$, and $h = (20, 20, 25)$. Since $Q(10) = 100 > 50 = Q(5) + Q(5)$, we have $f \succ g$, $g \succ h$, and $h \succ f$. Suppose that we improve the prospect h such that $h \succeq_{SD} f$, for instance let $h' = (25, 25, 25)$. As $f \succ g$ and, since Q is strictly increasing with h better than g across all states, we get $h' \succ f$ and $h' \succ g$, therefore transitivity holds. We can similarly show that if h is worsened such that $g \succ_{SD} h'$, then again transitivity holds. Thus, although the non-linearity of the Q function leads to intransitive preferences, the strictly increasing nature of Q function guarantees that regret theory satisfies dominance-transitivity. However, when $Q(x)$ is linear, then regret theory is equivalent to EU and, therefore, it is transitive for all prospects $f \in R^n$.

Comparing the behavioral foundation of regret theory with the behavioral foundation of rank-dependent utility (RDU) theories (Köbberling and Wakker, 2003), we observe that regret theory weakens the transitivity axiom of EU, whereas the RDU theories weaken the trade-off consistency axiom of EU. Thus both regret theory and RDU theories are able to accommodate the descriptive violations of EU by relaxing different axioms. Hence, when regret theory is transitive, its behavior is not consistent with RDU theories but only with EU.

Loomes and Sugden (1982) and Fishburn and LaValle (1988) argue for the normative status of regret theory. Loomes and Sugden (1982) argue that feelings of regret are a fact of life and it is irrational to ignore them, a view supported by Bourgeois-Gironde (2010) using neurodata. However, Bleichrodt and Wakker (2015) see no normative status for regret theory, but they stress its descriptive value. We next illustrate an application in the medical domain, where the descrip-

Table 4.1
Outcomes of Radiotherapy and Surgery in 3 years.

Treatments	s_1	s_2	s_3
Radiotherapy	Normal voice	Normal voice + death	Normal voice + death
Surgery	Artificial voice	Artificial voice	Artificial voice + death

tive power of regret theory is utilized for a prescriptive purpose. This application also illustrates the usefulness of our behavioral foundation.

4. Application: preference reversal in medical decision making

Consider a DM suffering from a stage T3 laryngeal cancer. T3 is a stage with no metastases (McNeil et al., 1981; Wakker, 2010). The choice between surgery and radiotherapy is difficult for stage T3, and there are no clear medical instructions favoring one treatment over the other. The advantage of surgery is that the treatment has fewer side effects and there is less chance of recurrence. The disadvantage is that the patient's voice is lost and the patient has to live with artificial voice. The advantage of radiotherapy is that the patient retains a normal voice, but the disadvantage is that the chance of recurrence is high (McNeil et al., 1981; Wakker, 2010). If the cancer recurs, the patient has less than 3 years to live. The best treatment decision — radiotherapy or surgery — depends on the variant of laryngeal cancer and that variant cannot be known beforehand. Therefore, doctors depend on patient's subjective preference in such cases.

The decision problem can be formulated by considering three states of nature: state s_1 – when both radiotherapy and surgery prevent recurrence, state s_2 – when only surgery prevents recurrence, and state s_3 – when both surgery and radiotherapy cannot prevent recurrence. The DM has to choose between radiotherapy and surgery. The decision problem is depicted in Table 4.1, adapted from Wakker (2010).

Consider a decision analyst aiding the DM to choose between radiotherapy and surgery. The decision analyst can either (i) elicit and compare the certainty equivalent of the individual treatments or (ii) formulate the decision problem as in Table 4.1 — with probabilities of the states specified — and ask the DM to choose between the treatments. Assume that the DM is regret averse and that preferences are well described by Eq. (1.1) with $Q(\alpha) = \alpha^3$, and with utilities for different outcomes normalized as follows: u (full health with no disease) $= 1 > u$ (normal voice) $= 0.7 > u$ (artificial voice) $= 0.5 > u$ (normal voice + death) $= 0.28 > u$ (artificial voice + death) $= 0.27 > u$ (immediate death) $= 0$. Note that the utilities and the regret function (Q) are not known a priori to the analyst or the DM. The probability of the states are $p(s_1) = 0.3$, $p(s_2) = 0.3$, and $p(s_3) = 0.4$.

Under the above assumptions, we show that the regret averse DM exhibits the classic preference reversal (Lichtenstein and Slovic, 1971): the DM displays a higher certainty equivalent for radiotherapy than for surgery when certainty equivalents are elicited, but prefers surgery over radiotherapy when making a choice between the treatments. The treatments can be represented in terms of prospect as follows: radiotherapy $= 0.7_{0.3}0.28$ and surgery $= 0.5_{0.6}0.27$. The certainty equivalent of radiotherapy is the sure utility amount β for which the DM is indifferent to the prospect $0.7_{0.3}0.28$, i.e., $\beta \sim 0.7_{0.3}0.28$. Applying the regret theory representation of Eq. (1.1) we get $\beta = 0.46$. Similarly, the certainty equivalent of surgery (β') is $\beta' \sim 0.5_{0.6}0.27$, which implies $\beta' = 0.39$ (by Eq. (1.1)). Therefore, CE (radiotherapy) $= 0.46 > 0.39 =$ CE (surgery). However, when the DM chooses between surgery and radiotherapy, by applying the regret theory representation in Eq. (1.1), we get $0.3 \times (0.5-0.7)^3 + 0.3 \times (0.5-0.28)^3 + 0.4 \times (0.27-0.28)^3 > 0$ which implies surgery $\succ$ radiotherapy. Therefore, the regret averse subject exhibits the classic preference reversal: CE (radiotherapy) > CE (surgery), but surgery $\succ$ radiotherapy.

To help the DM make a rational decision in such cases, the decision analyst can make a prescriptive use of our behavioral foundation as follows:

1. The axioms of our behavioral foundation help testing and verifying whether the DM behaves consistently with regret theory and, thereby, understanding if regret aversion is the reason for preference reversal.
2. If the DM behaves consistently with regret theory, then our behavioral foundation allows using trade-off consistency to elicit utilities independently of regret (see Appendix A – Stage 1 of Bleichrodt et al., 2010).[10]
3. Once the utility function has been elicited, by fixing extreme outcomes (death and normal voice) as gauge, the analyst can elicit the values of the regret function (Q) function (see Appendix A – Stage 2 of Bleichrodt et al., 2010).
4. Finally, the analyst can use the utilities — debiased from regret — to estimate expected utility for the DM and help him making a rational decision. In this case, under expected utility, radiotherapy $\succ$ surgery.

5. Conclusion

This paper introduces a new foundation for regret theory. Our foundation is consistent with a recently introduced measurement technique for regret theory. The behavioral foundation is the first to allow for a continuous regret theory representation and to separate "rational" utility from regret. The axioms of the behavioral foundation are natural and transparent, and allow clarifying the relationship between regret theory and expected utility. In particular, they capture that the only difference between expected utility and regret theory lies in abandoning transitivity. The paper technically contributes to the literature by providing new insights into the relationship between (i) trade-off consistency and preference trade-off consistency (Krantz et al., 1971), and (ii) preference trade-off consistency and the independence axiom in Fishburn (1990). The paper is also the first to axiomatize a non-transitive theory using trade-off consistency. Thus, more than three decades after the publication of Loomes and Sugden's path-breaking work, we offer a complete foundation that exactly identifies the normative and descriptive appeal of regret theory.

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[10] However, one challenge in applying trade-off consistency (and our behavioral foundation) to the problem above is that the outcome space has to be rich. This requires the analyst to consider a rich set of health outcomes: For example one might start with the lowest outcome – death in 3 years and then consider subsequent outcomes such as the death in 3 years + little bit of artificial voice ($\epsilon \to 0$ artificial voice) and so on until one reaches the outcome "normal voice," which has the highest utility.

Appendix A

The two stages of the Bleichrodt, Cillo, and Diecidue (2010) method to measure utility and regret functions, are described next.

Stage 1

The first stage replicates Wakker and Deneffe (1996) and enables the measurement of the utility function under regret theory. In details: The subject is asked to choose the outcome x_1 that would make him indifferent between the prospects $x_{1_p}g_1$ (receiving x_1 with probability p under s_1 and g_1 otherwise) and $x_{0p}g_2$; here g_1, g_2 are fixed "gauge" outcomes, x_0 is the starting outcome (fixed) and $p \in [0, 1]$ is a given (fixed) probability. The method is based on a choice task: the subject is asked to choose between two prospects $x_{1p}g_1$ and $x_{0p}g_2$ for different values of x_1 until indifference is reached (see Appendix B for an illustration of this choice task).

Once x_1 is elicited, the procedure, similarly elicits from the subject an outcome x_2 such that the subject is indifferent between the prospects $x_{2p}g_1$ and $x_{1p}g_2$. Repeated application of the same procedure allows to elicit a sequence of outcomes $x_0, ..., x_k$. For the last outcome elicited x_k, $u(x_k)$ is scaled to 1 and $u(x_0)$, the utility of the first outcome of the sequence, is scaled to 0. The outcomes $x_0, ..., x_k$ are equally spaced in utilities (i.e., $u(x_i) = 1/k$ for all $i \leq k$); they are called a standard sequence of outcomes. To smooth out irregularities, the standard sequence of outcomes can be fitted with a parametric utility function — for example, a power function of the form $u(\alpha) = \alpha^\theta$ — to build the utility function u under regret theory.

Stage 2

The second stage of the method measures the regret function Q. The outcomes z_j are elicited such that the subject is indifferent between the prospects $x_{kp_j}x_0 \sim x_{k-1p_j}z_j$, where x_k and x_{k-1} are (respectively) the last and the penultimate outcomes from the standard sequence. As in stage 1, the values of z_j are elicited using a choice task and are elicited for different probabilities p_j. The utility of z_j is then calculated by linear interpolation of the standard sequence of outcomes. The value of the Q function is computed via $Q(u(z_j)) = p_j/(1 - p_j)$ and by scaling $u(x_0) = 0$, $u(x_k) = 1$ and $Q(x_k - x_{k-1} = 1/k) = 1$, where k is the total number of standard sequence outcomes elicited. To smooth out irregularities, the values of Q can again be fitted with a power function of form $Q(\alpha) = \alpha^\theta$ to build the regret function Q. To sum up: the method measures both the utility function (u) and the regret function (Q) at the individual level. This method was introduced and validated by an experiment reported in Bleichrodt, Cillo, and Diecidue (2010).

Appendix B

Choice task

We present the choice task (bisection) following Bleichrodt et al. (2010). In the measurement of u, x_{j+1} was elicited through choices between $x = x_{j_p}g_1$ and $y = x_{j+1_p}g_2$, $j = 0, ..., 4$. The initial value of x_{j+1} was a random integer in the interval $[x_j, x_j + 5 \cdot (g_1 - g_2)]$. There were two possible scenarios: (i) If x was chosen then x_{j+1} was increased by $D = 5 \cdot (g_1 - g_2)$ until y was chosen; then x_{j+1} was decreased by $D/2$. If x (resp., y) was subsequently chosen then x_{j+1} was increased (resp., decreased) by $D/4$ and so forth. (ii) If y was chosen, x_{j+1} was decreased

Table B.1
Example of bisection method ($p = 0.5$).

Iteration	x_1	Choice
1	26	x
2	46	y
3	36	y
4	31	x
5	34	x

by $D = (x_{j+1} - x_j)/2$ until x was chosen then x_{j+1} increased by $D/4$. If x was subsequently chosen, then x_{j+1} was increased (decreased) by $D/8$ and so forth. The elicitation was stopped when the difference — between the lowest value of x_{j+1} for which y was chosen and the highest value of x_{j+1} for which x was chosen — was no more than 2. The recorded indifference was the midpoint between two values. Table B.1 presents an example of the procedure for eliciting x_1 through comparison between $x = (20)_{0.5}17$ and $y = (x_1)_{0.5}13$. In this example, the initial random value of x_1 was 26. The indifference value was 35 (i.e., the midpoint between 34 and 36). The bisection method just described for stage 2 is similar.

Appendix C

State-dependent regret representation (Fishburn, 1990)

To introduce Fishburn's (1990) representation, additional notation is needed. Consider a positive integer n and $\mathbb{N} = \{1, \ldots, n\}$. Let X_i be a non-empty set. $(X_i, \mathbb{H}_i)$ is a topological space for each $i \in \mathbb{N}$. Let $X = X_1 \times \ldots \times X_n = \times X_i$ and $T_i = X_i \times X_i$ with product topology $\mathcal{F}_i = \mathbb{H}_i \times \mathbb{H}_i$. Also let $t_i \in T_i$.

Definition 6. Fishburn's (1990) representation holds, if there is a continuous skew-symmetric function $\psi_i : T_i \to \mathbb{R}$ such that

$$f \succeq g \Longleftrightarrow \sum_{i=1}^{n} \psi_i(t_i) \geq 0 \Longleftrightarrow \sum_{i=1}^{n} \psi_i(f_i, g_i) \geq 0; \tag{C.1}$$

here $t_i = (f_i, g_i)$, $f = (f_1, \ldots, f_n)$ and $g = (g_1, \ldots, g_n)$ are prospects.

Note that Definition 6 is close to a regret theory representation. The main difference is that in the regret theory representation of Eq. (2.1) there is a state independent ψ function separated from subjective probabilities p_i, while in Eq. (C.1) the ψ_i functions are not separated from subjective probabilities. For this reason, the representation in Definition 6 is to be considered a state-dependent regret representation. Now before we introduce the axioms of Fishburn (1990), we will introduce some additional notation.

The diagonal of T_i is $D_i = \{(f_i, f_i) : f_i \in X_i\}$. Let $T = \times T_i$ and $D = \times D_i$ with members $t = (t_1, \ldots, t_n)$ and $d = (d_1, \ldots, d_n)$, respectively. The inverse of $t_i = (f_i, g_i)$ in T_i is $t_i^{-1} = (g_i, f_i)$, and the inverse of $t \in T$ is $t^{-1} = (t_1^{-1}, \ldots, t_n^{-1})$. Note that $d^{-1} = d$ for all $d \in D$. For convenience, set

$$T_{(i)} = \times_{j \neq i} T_j \text{ and } D_{(i)} = \times_{j \neq i} D_j$$

and with the customary abuse of notation write $t \in T$ as (t_i, w) or (w, t_i) when $w = (t_1, \ldots t_{i-1}, t_{i+1}, \ldots, t_n)$ is in $T_{(i)}$. We also let T_J denote the product of the T_i for $i \in J$, with $t_J \in T_J$. The product of T with itself m times is T^m.

Define $F_m \subseteq T^m$ by $(t^1, \ldots, t^m) \in F_m$ if $t^j \in T$ for all $j \in M$ and for all $i \in \mathbb{N}$ and all $\alpha \in T_i$, $|\{j : t_i^j = \alpha\}| = |\{j : t_i^j = \alpha^{-1}\}|$.[11]

This says that, for each factor T_i, the ith components of the t^j are precisely matched by inverse pairs. When each ψ_i is a skew symmetric functional on T_i,

$$(t^1, \ldots, t^m) \in F_m \Longrightarrow \sum_{i,j} \psi_i(t_i^j) = 0.$$

The F_m relations are the basis of our ensuing discussion of independence among factors. Finally let P denote the subset of T, with inverse P^{-1}, and let $I = T \backslash (P \cup P^{-1})$, the symmetric complement of P in T. Clearly, $I^{-1} = I$. In this sub-section, we represent $(f_1, \ldots, f_n) \succ (g_1, \ldots, g_n)$ by $((f_1, g_1), \ldots, (f_n, g_n)) \in P$, with I the correspondent of the usual indifference relation $\sim$ on X.

Now we introduce the Fishburn (1990) axioms.

(a) Structure. $n \geq 3$, $(X_i, \mathbb{H}_i)$ is a connected and separate topological space for each $i \in \mathbb{N}$ and for each $i \in \mathbb{N}$ there exist $t_i \in T_i$ and $w \in D_{(i)}$ such that $(t_i, w) \in P$.

The reason for $n \geq 3$ in the Axiom (a) is that, for $n = 2$ there is insufficient structure in the non-transitive setting for 'nice' representation and uniqueness theorems (for discussion see Fishburn, 1990, page 170). Since our outcome set is $\mathbb{R}$, the first part of the structure axiom is automatically satisfied. The final part of the structure axiom is a non-triviality condition which ensures that each factor plays an active role in the representation.

(b) Continuity. For all $i \in \mathbb{N}$ and all $w \in T_{(i)}$, $\{t_i \in T_i : (t_i, w) \in P\} \in \mathcal{F}_i$.

Continuity is a technical axiom that ensures that small changes in the outcomes do not affect the preference.

(c) Independence. For all $(t^1, t^2, t^3, t^4) \in F_4$, if $t^j \in P \cup I$ for $j = 1, 2, 3$, then $t^4 \notin P$.

The independence axiom is key in deriving the Fishburn (1990) representation. Though it is by itself sufficient to derive a state-dependent regret representation (in the presence of technical axioms a, b and d), it is complex to understand. We give the intuition of the independence axiom below by considering the case of two outcome prospects. The independence axiom states that, for two-outcome prospects, if (i) $(\alpha_1', \beta_1) \succeq (\alpha_2', \beta_2)$, (ii) $(\alpha_1, \beta_2) \succeq (\alpha_2, \beta_1)$, and (iii) $(\alpha_2, \beta_1') \succeq (\alpha_1, \beta_2')$ hold, then (iv) $(\alpha_2', \beta_2') \not\succ (\alpha_1', \beta_1')$ holds. To understand the independence axiom, we will focus our attention on F_m, with $m = 4$, that is F_4. The four set of prospects are from F_4 if each factor T_i is matched by its inverse, i.e., $t_1^1 = (\alpha_1', \alpha_2')$ is in (i) and the inverse of $(t_1^1)^{-1} = t_1^4 = (\alpha_2', \alpha_1')$ is in (iv). Similarly $t_1^2 = (t_1^3)^{-1} = (\alpha_1, \alpha_2)$. So for $i = 1$, we have, $|\{j : t_i^j = \alpha\}| = |\{j : t_i^j = \alpha^{-1}\}|$. Similarly we have $|\{j : t_i^j = \alpha\}| = |\{j : t_i^j = \alpha^{-1}\}|$ for $i = 2$. In other words,

[11] Fishburn (1990) refers to F_m using the E_m notation.

this says that, prospects are in F_4 if for each factor T_i, the ith components of the t^j are precisely matched by inverse pairs.

Now we will discuss the independence axiom. The independence axiom is very similar to preference trade-off consistency. Both preference trade-off consistency and the independence axiom implies that if (i), (ii) and (iii) hold, then (iv) holds. For the case of a two-outcome prospect, preference trade-off consistency restricted to a particular state coincides with the independence axiom. However the independence axiom begins to deviate from preference trade-off consistency for three or more outcome prospects, especially when the set of prospects do not have a common gauge outcome. For example, the following set of prospects (i) $(\alpha_1', \beta_1, \gamma_1) \succeq (\alpha_2', \beta_2, \gamma_2)$ (ii) $(\alpha_1, \beta_2, \gamma_1') \succeq (\alpha_2, \beta_1, \gamma_2')$ (iii) $(\alpha_2, \beta_1', \gamma_2) \succeq (\alpha_1, \beta_2', \gamma_1)$ (iv) $(\alpha_2', \beta_2', \gamma_2') \succ (\alpha_1', \beta_1', \gamma_1')$ satisfies F_4 and violates Fishburn independence. However preferences above do not violate preference trade-off consistency directly because there is no common gauge outcomes, for instance, the outcomes β_1, γ_1 and β_2, γ_2 in (i) do not match the outcomes in states 2 and 3 of prospects in (ii), (iii), and (iv). As a result, we cannot apply Eq. (2.5) and preference trade-off consistency is not directly violated. We discuss this case in detail in our proof (see Lemma 4).

(d) Nonextremality. For all $i \in \mathbb{N}$ and all $t_i \in T_i$, there is a $w \in T_{(i)}$ such that $(t_i, w) \notin P$.

The non-extremality axiom restricts the value of a particular factor T_i from being extreme. This property allows Fishburn's (1990) representation to be unique up to a proportionality transformation.

Now we state the representation theorem of Fishburn (1990).

Theorem 2. *Suppose Axioms (a) through (d) hold. Then for each $i \in \mathbb{N}$ there is a continuous skew symmetric functional ψ_i on T_i such that*

$$f \succ g \Longleftrightarrow \sum_{i=1}^{n} \psi_i(f_i, g_i) > 0\,;$$

here $f = (f_1, \ldots, f_n)$ and $g = (g_1, \ldots, g_n)$ are prospects and the ψ_i are unique up to similarity proportional transformations.

Appendix D

Proof of Theorem 1

Lemma 1. *If regret theory holds with a strictly increasing utility function u and a strictly increasing, skew symmetric regret function Q, then the preference relation $\succeq$ satisfies completeness, d-transitivity, strong monotonicity, continuity and trade-off consistency.*

Proof. *i. Completeness:* For any two prospects $f = (f_1, \ldots, f_n)$ and $g = (g_1, \ldots, g_n) \in \mathbb{R}^n$, if regret theory holds, the prospects are evaluated by the function $\sum_{i=1}^n p_i(Q(u(f_i) - u(g_i))$, which can be either > 0 or < 0 or $= 0$, hence $f \succeq g$ or $g \succeq f$ should hold. Therefore the preference relation $\succeq$ satisfies completeness.

ii. D-transitivity: For prospects $f = (f_1, \ldots, f_n)$, $g = (g_1, \ldots, g_n)$, and $h = (h_1, \ldots, h_n)$, if $g \succeq h$, then $\sum_{i=1}^n p_i \cdot Q(u(g_i) - u(h_i)) \geq 0$. If f strictly dominates g ($f \succeq_{SD} g$), then $\forall i$, $f_i \geq g_i$ and $\exists i$ such that $f_i > g_i$. Therefore as Q is strictly increasing, we get $\sum_{i=1}^n p_i \cdot Q(u(f_i) -$

$u(g_i)) > 0$, which along with $\sum_{i=1}^{n} p_i \cdot Q(u(g_i) - u(h_i)) \geq 0$ implies $\sum_{i=1}^{n} p_i \cdot Q(u(f_i) - u(h_i)) > 0$. Thus $f \succ h$ and the preference relation $\succeq$ satisfies d-transitivity.

iii. Strong monotonicity: For any two prospects $f = (f_1, \ldots, f_n)$ and $g = (g_1, \ldots, g_n) \in \mathbb{R}^n$, if $f_i \geq g_i, \forall i = 1, ..., n$, and $f_i > g_i$ for an state s_i, then $\sum_{i=1}^{n} p_i Q(u(f_i) - u(g_i)) > 0$ (as u and Q are strictly increasing). If regret theory holds then $\sum_{i=1}^{n} p_i \cdot Q(u(f_i) - u(g_i)) > 0$ implies $f \succ g$, and therefore strong monotonicity holds.

iv. Continuity: For all $f, g \in \mathbb{R}^n$, as the utility function $u : \mathbb{R} \to \mathbb{R}$ and the regret function $Q : \mathbb{R} \to \mathbb{R}$ are continuous, the sets $\{(f_1, \ldots, f_n) : (f_1, \ldots, f_n) \succeq (g_1, \ldots, g_n)\}$ and $\{(f_1, \ldots, f_n) : (f_1, \ldots, f_n) \preceq (g_1, \ldots, g_n)\}$ are closed, thereby the preference relation $\succeq$ satisfies continuity.

v. Trade-off (TO) Consistency: Consider the following relationship between the prospects, $\alpha_i w \sim \beta_i x$, $\gamma_i w \sim \delta_i x$ and $\alpha'_j y \sim \beta_j z$, $\gamma_j y \sim \delta_j z$, where $\alpha, \alpha', \beta, \gamma$, and δ are outcomes that replace the outcome contingent on state s_i or s_j in each of the prospects w, x, y, and $z \in \mathbb{R}^n$. If regret theory holds then $p_i Q(u(\alpha) - u(\beta)) = -\sum_{k \neq i} p_k Q(u(w_k) - u(x_k)$, $p_i Q(u(\gamma) - u(\delta)) = -\sum_{k \neq i} p_k Q(u(w_k) - u(x_k))$, so we get $Q(u(\alpha) - u(\beta)) = Q(u(\gamma) - u(\delta)) \Leftrightarrow \alpha \ominus \beta \sim_t \gamma \ominus \delta$. Similarly, we get $Q(u(\alpha') - u(\beta)) = Q(u(\gamma) - u(\delta)) \Leftrightarrow \alpha' \ominus \beta \sim_t \gamma \ominus \delta$. From the two equalities we get, $Q(u(\alpha) - u(\beta)) = Q(u(\alpha') - u(\beta))$. Since Q and u are strictly increasing, $Q(u(\alpha) - u(\beta)) = Q(u(\alpha') - u(\beta)) \Longrightarrow \alpha = \alpha'$. Hence the preference relation $\succeq$ satisfies TO consistency. □

Assumption 1. *Completeness, d-transitivity, continuity, strong monotonicity and trade-off consistency holds.*

Lemma 2. *Under Assumption 1, $\alpha_i f \sim \alpha_i g \Longleftrightarrow \beta_i f \sim \beta_i g$.*

Proof. If state s_i is null, then changing outcome in state s_i does not affect the preference. So $\alpha_i f \sim \alpha_i g \Longleftrightarrow \beta_i f \sim \beta_i g$ is satisfied. Next if state s_i is not null, then by TO consistency $\alpha_i f \sim \alpha_i f$, $\beta_i f \sim \beta_i f$ and $\alpha_i f \sim \alpha_i g$ imply $\beta_i f \sim \beta_i g$. The reverse implication follows similarly. □

Proposition 1. *Under Assumption 1, preference trade-off consistency holds.*

Proof. We derive the final preparatory lemma within this proof. The end of proofs within proof are indicated by QED and not by □.

Lemma 3. *Assume strong monotonicity, d-transitivity, and continuity. Let $\alpha_i f \succeq \beta_i g$ and $\gamma_i f \preceq (\prec)\delta_i g$, where all four acts are contained in $\mathbb{R}^n$. Then there exists $\bar{f}, \bar{g}$ with ($\alpha_i \bar{f} \sim \beta_i \bar{g}$ and $\gamma_i \bar{f} \preceq (\prec)\delta_i \bar{g}$) or ($\alpha_i \bar{f} \succ \beta_i \bar{g}, \gamma_i \bar{f} \preceq (\prec)\delta_i \bar{g}$ and $\bar{f}_j \leq \bar{g}_j$ for all $j \neq i$). In the latter case $\alpha > \beta$. Furthermore in any case we can have $\bar{f}_j \leq f_j$ and $\bar{g}_j \geq g_j$ for all $j \neq i$.*

Proof. Let $\alpha_i f, \beta_i g, \gamma_i f, \delta_i g \in \mathbb{R}^n$ be such that $\alpha_i f \succeq \beta_i g$ and $\gamma_i f \preceq (\prec)\delta_i g$.

Step 1. We will push the g_j with $g_j < f_j$ up towards f_j, and next push the f_j with $f_j > g_j$ down until either an indifference $\alpha_i f \sim \beta_i g$ results, or $f_j \leq g_j$ for all $j \neq i$ and still $\alpha_i f \succ \beta_i g$ (then α must be much better than β otherwise strong monotonicity would be violated).

Step 2: Because there are only finitely many states, the procedure in Step 1 ends after finitely many repetitions. Define $\bar{f}$ and $\bar{g}$ as the newly constructed acts f and g from step 1. We have either $\alpha_i \bar{f} \sim \beta_i \bar{g}$ or [$\alpha_i \bar{f} \succ \beta_i \bar{g}$ and $\bar{f}_j \preceq \bar{g}_j$ for all $j \neq i$]. In what follows f and g refer again to the original acts f and g. We know that by strong monotonicity $\bar{f} \preceq f$. Then $\gamma_i \bar{f} \preceq \gamma_i f$. For the act g similarly by strong monotonicity we know that $\bar{g} \succeq g$. Then $\delta_i \bar{g} \succeq \delta_i g$. Now we have

$\gamma_i \bar{f} \preceq \gamma_i f \preceq (\prec)\delta_i g \preceq \delta_i \bar{g}$. By d-transitivity axiom we get $\delta_i \bar{g} \succeq (\succ)\gamma_i f \succeq \gamma_i \bar{f}$. Again applying d-transitivity we get $\delta_i \bar{g} \succeq (\succ)\gamma_i \bar{f}$. This completes the proof of Lemma 3. QED.

Now the rest of the proof of Proposition 1 follows. Assume that there are prospects

$$\alpha_i f \succeq \beta_i g,\ \gamma_i f \preceq \delta_i g,\ \alpha_j x \preceq \beta_j y,\ \gamma_j x \succ \delta_j y$$

such that states s_i and s_j are non-null.

Step 1. In this step, we show that either $\alpha \leq \gamma$ and $\beta \leq \delta$ or $\alpha \geq \gamma$ and $\beta \geq \delta$. Assume $\alpha < \gamma$. By strong monotonicity this implies $\delta_i g \succeq \gamma_i f \succ \alpha_i f \succeq \beta_i g$. By d-transitivity we get $\delta_i g \succ \alpha_i f \succeq \beta_i g$. Suppose $\delta_i g \preceq \beta_i g$, then $\delta \leq \beta$, which gives $\beta_i g \succeq \delta_i g \succ \alpha_i f$, by d-transitivity, this would imply $\beta_i g \succ \alpha_i f$. This is a contradiction as we know already that $\alpha_i f \succeq \beta_i g$. So $\delta_i g \succ \beta_i g$. Again by strong monotonicity we get $\delta > \beta$. Next assume $\alpha \geq \gamma$. Then similarly $\beta_j y \succeq \alpha_j x \succeq \gamma_j x \succ \delta_j y$ and $\beta > \delta$.

Step 2. In this step, we show that there exist $\bar{\alpha}_i f \sim \bar{\beta}_i g$, $\bar{\gamma}_i f \sim \bar{\delta}_i g$, $\bar{\alpha}_j x \prec \bar{\beta}_j y$ and $\bar{\gamma}_j x \succ \bar{\delta}_j y$. In each of the two cases: $\alpha \leq \gamma$ and $\beta \leq \delta$ or $\alpha \geq \gamma$ and $\beta \geq \delta$, $\bar{\alpha}$ and $\bar{\gamma}$ are between α and γ, and $\bar{\beta}$ and $\bar{\delta}$ are between β and δ, in preference. First, assume $\alpha \leq \gamma$ and $\beta \leq \delta$. We have $\delta_i g \succeq \gamma_i f \succeq \alpha_i f \succeq \beta_i g$. By d-transitivity we get $\delta_i g \succeq \alpha_i f \succeq \beta_i g$ and $\delta_i g \succeq \gamma_i f \succeq \beta_i g$. Continuity implies that there is a $\bar{\beta}$ with $\alpha_i f \sim \bar{\beta}_i g$ and a $\bar{\delta}$ with $\gamma_i f \sim \bar{\delta}_i g$. We can get $\delta \geq \bar{\delta} \geq \bar{\beta} \geq \beta$, from which it follows that $\alpha_j x \preceq \bar{\beta}_j y$ and $\gamma_j x \succ \bar{\delta}_j y$ (by d-transitivity). In this case, let $\bar{\alpha} = \alpha$ and $\bar{\gamma} = \gamma$. We must have $\bar{\alpha}_j x \prec \bar{\beta}_j y$, because the other case, $\bar{\alpha}_j x \sim \bar{\beta}_j y$, would imply by trade-off consistency that $\bar{\gamma}_j x \sim \bar{\delta}_j y$, which is a contradiction.

Second, assume $\alpha \geq \gamma$ and $\beta \geq \delta$. Similarly to the previous case continuity implies that there is an $\bar{\alpha}$ with $\alpha \geq \bar{\alpha} \geq \gamma$ and $\bar{\alpha}_i f \sim \beta_i g$, and a $\bar{\gamma}$ with $\bar{\alpha} \geq \bar{\gamma} \geq \gamma$ and $\bar{\gamma}_i f \sim \delta_i g$. Furthermore $\bar{\alpha}_j x \preceq \beta_j y$ and $\bar{\gamma}_j x \succ \delta_j y$. In this case, let $\bar{\beta} = \beta$ and $\bar{\delta} = \delta$. By trade-off consistency we must have $\bar{\alpha}_j x \prec \bar{\beta}_j y$.

As a preparation for the following step, we rename for notational convenience, $\alpha = \bar{\alpha}$, $\beta = \bar{\beta}$, $\gamma = \bar{\gamma}$ and $\delta = \bar{\delta}$, so that we have $\alpha_i f \sim \beta_i g$, $\gamma_i f \sim \delta_i g$, $\alpha_j x \prec \beta_j y$, $\gamma_j x \succ \delta_j y$. Furthermore, as before, either $\alpha \leq \gamma$ and $\beta \leq \delta$, or $\alpha \geq \gamma$ and $\beta \geq \delta$. Because of symmetry, we may assume hereafter $\alpha \leq \gamma$ and $\beta \leq \delta$.

Step 3. We will rule out all cases except $\gamma > \delta \geq \beta > \alpha$. By Lemma 3, we can find $\bar{x}$, $\bar{y}$ (pushing the $x_j s$ up and $y_j s$ down) such that either $\alpha_j \bar{x} \sim \beta_j \bar{y}$ and $\gamma_j \bar{x} \succ \delta_j \bar{y}$ or $\alpha_j \bar{x} \prec \beta_j \bar{y}$ and $\gamma_j \bar{x} \succ \delta_j \bar{y}$ with $x_i \geq y_i$ $\forall i \neq j$. The first case leads to contradiction of trade-off consistency. Therefore we assume that the second case holds. This implies, in particular, $\alpha < \beta$ because $x_i \geq y_i$ for all $i \neq j$.

Again by Lemma 3, we can find $\bar{x}$, $\bar{y}$ (pushing the $x_j s$ down and $y_j s$ up) such that either $\gamma_j \bar{x} \sim \delta_j \bar{y}$ and $\alpha_j \bar{x} \prec \beta_j \bar{y}$ or $\gamma_j \bar{x} \succ \delta_j \bar{y}$ and $\alpha_j \bar{x} \prec \beta_j \bar{y}$ with $x_i \leq y_i, \forall i \neq j$. The first half leads to contradiction because of Lemma 3. We therefore assume that the second case holds. This implies in particular, $\gamma > \delta$, because $\bar{x_i} \leq \bar{y}_i, \forall i \neq j$.

Step 4. We have $\gamma > \delta \geq \beta > \alpha$. This will lead to the final contradiction. Strong monotonicity implies $\beta_i f \succ \alpha_i f \sim \beta_i g$ and $\delta_i f \prec \gamma_i f \sim \delta_i g$. By d-transitivity, we get $\beta_i f \succ \beta_i g$ and $\delta_i f \prec \delta_i g$. Now Lemma 3 implies that we can find $\bar{f}, \bar{g}$ such that $\beta_i \bar{f} \sim \beta_i \bar{g}$ and $\delta_i \bar{f} \prec \delta_i \bar{g}$ (with $\bar{f}_j \leq f_j$ and $\bar{g}_j \geq g_j$ for all $j \neq i$), because the second case in that lemma implying the impossible $\beta > \beta$, cannot occur here. A contradiction with Lemma 2 has resulted. Thus Proposition 1 is proved. □

Lemma 4. *If completeness, d-transitivity, continuity and strong monotonicity hold, then preference trade-off consistency implies Fishburn's (1990) independence axiom.*

Proof. Our approach to the proof is as follows. First for the two-outcome prospects (with two states of nature), we show that under the assumptions in Lemma 4, preference trade-off consistency implies Fishburn's (1990) independence axiom. Then we extend the implication to three-outcome prospects with and without common gauge. Finally, by mathematical induction we extend the implication to all n-outcome prospects (with n states of nature). In the proof below, for convenience, we use a Greek letter with subscripts: $\alpha_i, \alpha_i', \beta_i, \beta_i', \gamma_i, \gamma_i', \delta_i$ and δ_i', to indicate the outcomes of a prospect.

Step 1: Consider two-outcome prospects with the following preferences. Assume for a contradiction that the preferences satisfy Fishburn's (1990) F_4 condition but violates Fishburn independence: (i) $(\alpha_1', \beta_1) \succeq (\alpha_2', \beta_2)$, (ii) $(\alpha_1, \beta_2) \succeq (\alpha_2, \beta_1)$, (iii) $(\alpha_2, \beta_1') \succeq (\alpha_1, \beta_2')$ and (iv) $(\alpha_2', \beta_2') \succ (\alpha_1', \beta_1')$. From (i) and (ii) we get $\alpha_1'\alpha_2' \succeq_t \alpha_2\alpha_1$, from (iii) and (iv) we get $\alpha_2\alpha_1 \succ_t \alpha_1'\alpha_2'$. Thus preference trade-off consistency is violated and the implication in Lemma 4 is proved. Now we consider one other combination in a two-outcome prospect for which F_4 is satisfied but Fishburn independence is violated, i.e., we replace β_2, β_1 in (ii) with β_1', β_2' in (iii) then we get (i) $(\alpha_1', \beta_1) \succeq (\alpha_2', \beta_2)$, (ii) $(\alpha_1, \beta_1') \succeq (\alpha_2, \beta_2')$, (iii) $(\alpha_2, \beta_2) \succeq (\alpha_1, \beta_1)$ and (iv) $(\alpha_2', \beta_2') \succ (\alpha_1', \beta_1')$. Now we find $\alpha_1'\alpha_2' \succeq_t \alpha_1\alpha_2$ (from (i) and (iii)) and $\alpha_1\alpha_2 \succ_t \alpha_1'\alpha_2'$ (from (ii) and (iv)), again this violates preference trade-off consistency and the implication is proved. There are other combinations like replacing, α_2, α_1 in (ii) with α_1', α_2' in (iv) or α_2, α_1 in (iii) with α_1', α_2' in (i), that would satisfy F_4 and violate Fishburn independence. In all these cases preference trade-off consistency will be violated. Similarly we can show that for all combinations in which α_1, α_2 are interchanged with α_2', α_1' and sub cases for which $\beta_1 = \beta_1', \beta_2 = \beta_2'$ or $\alpha_1 = \alpha_1', \alpha_2 = \alpha_2'$, preference trade-off consistency implies Fishburn independence under the assumptions of Lemma 4. Hence we have shown that for a two-outcome prospect, preference trade-off consistency implies Fishburn independence under the assumptions of Lemma 4.

Step 2a: Now we extend the implication for three-outcome prospects with common gauge. Consider the following preferences among three-outcome prospects with a common gauge that satisfies F_4 but violates Fishburn independence: (i) $(\alpha_1', \beta_1, \gamma_1) \succeq (\alpha_2', \beta_2, \gamma_2)$ (ii) $(\alpha_1, \beta_2, \gamma_2) \succeq (\alpha_2, \beta_1, \gamma_1)$ (iii) $(\alpha_2, \beta_1', \gamma_1') \succeq (\alpha_1, \beta_2', \gamma_2')$ and (iv) $(\alpha_2', \beta_2', \gamma_2') \succ (\alpha_1', \beta_1', \gamma_1')$. From (i) and (ii), we get $\alpha_1'\alpha_2' \succeq_t \alpha_2\alpha_1$ and from (iii) and (iv), we get $\alpha_2\alpha_1 \succ_t \alpha_1'\alpha_2'$. Such preferences violate preference trade-off consistency and hence the implication is proved. Similarly, for other combinations in three-outcome prospects with common gauge, we can prove that, under the assumptions of Lemma 4, preference trade-off consistency implies Fishburn independence.

Step 2b: Now we extend the implication to a more complicated case, i.e., a three-outcome prospect without a common gauge. Consider the following preferences among three-outcome prospect without a common gauge that satisfies F_4 but violates Fishburn independence: (i) $(\alpha_1', \beta_1, \gamma_1) \succeq (\alpha_2', \beta_2, \gamma_2)$, (ii) $(\alpha_1, \beta_2, \gamma_1') \succeq (\alpha_2, \beta_1, \gamma_2')$, (iii) $(\alpha_2, \beta_1', \gamma_2) \succeq (\alpha_1, \beta_2', \gamma_1)$, (iv) $(\alpha_2', \beta_2', \gamma_2') \succ (\alpha_1', \beta_1', \gamma_1')$. Now our objective would be to show that such a preference is not possible. To show that we have to modify prospects (i), (ii), (iii), and (iv) in such a way that they will have a common gauge. Suppose we change γ_1' to γ_2 and γ_2' to γ_1 in (ii), and we change γ_2 to γ_1' and γ_1 to γ_2' in (iii) then the prospects in (i), (ii), (iii) and (iv) will have a common gauge. In other words we interchange outcomes γ_1' and γ_2' in (ii) with γ_2 and γ_1 in (iii) such that (i), (ii), (iii), and (iv) have a common gauge. However this interchanging could affect the preferences in (ii) and (iii). Below we show that for all possible change to preferences in (ii) and (iii), preference trade-off consistency is violated. Before we discuss the individual cases, note from preferences in step 1 and from preference trade-off consistency (a) $(\alpha_1', \beta_1, \gamma_1) \succeq (\alpha_2', \beta_2, \gamma_1)$, (b) $(\alpha_1, \beta_2, \gamma_1) \succeq (\alpha_2, \beta_1, \gamma_1)$, (c) $(\alpha_2, \beta_1', \gamma_1) \succeq (\alpha_1, \beta_2', \gamma_1)$ and (d) $(\alpha_2', \beta_2', \gamma_1) \not\succ (\alpha_1', \beta_1', \gamma_1)$

holds for any outcome $\gamma_1 \in \mathbb{R}$. Now we discuss the possible cases because of changing γ_1' to γ_2, γ_2' to γ_1 in (*ii*), and γ_2 to γ_1' and γ_1 to γ_2' in (*iii*)

Case 1: There is no change in preferences of (*ii*) and (*iii*) We get (*i*) $(\alpha_1', \beta_1, \gamma_1) \succeq (\alpha_2', \beta_2, \gamma_2)$, (*ii*) $(\alpha_1, \beta_2, \gamma_2) \succeq (\alpha_2, \beta_1, \gamma_1)$, (*iii*) $(\alpha_2, \beta_1', \gamma_1') \succeq (\alpha_1, \beta_2', \gamma_2')$, (*iv*) $(\alpha_2', \beta_2', \gamma_2') \succ (\alpha_1', \beta_1', \gamma_1')$. From (*i*) and (*ii*), we get $\alpha_1'\alpha_2' \succ_t \alpha_2\alpha_1$. From (*iii*) and (*iv*) we get $\alpha_2\alpha_1 \succ_t \alpha_1'\alpha_2'$. Such preferences violate preference trade-off consistency and the implication is proved for this case.

Case 2: There is a change in preference of (*ii*) but no change in (*iii*). We get (*i*) $(\alpha_1', \beta_1, \gamma_1) \succeq (\alpha_2', \beta_2, \gamma_2)$, (*ii*) $(\alpha_1, \beta_2, \gamma_2) \preceq (\alpha_2, \beta_1, \gamma_1)$, (*iii*) $(\alpha_2, \beta_1', \gamma_1') \succeq (\alpha_1, \beta_2', \gamma_2')$ (*iv*) $(\alpha_2', \beta_2', \gamma_2') \succ (\alpha_1', \beta_1', \gamma_1')$. Note that such preference is possible only if $\gamma_1 > \gamma_2$. Otherwise if $\gamma_2 > \gamma_1$, by strong monotonicity and preference in (*b*), we get $(\alpha_1, \beta_2, \gamma_2) \succ (\alpha_1, \beta_2, \gamma_1) \succeq (\alpha_2, \beta_1, \gamma_1)$, and by d-transitivity we will get $(\alpha_1, \beta_2, \gamma_2) \succ (\alpha_2, \beta_1, \gamma_1)$, which contradicts the preference in (*ii*) So γ_1 should exceeds γ_2.

Now we increase γ_2 to $\gamma_2 + \triangle$ such that the preference in (*ii*) becomes $(\alpha_1, \beta_2, \gamma_2 + \triangle) \succeq (\alpha_2, \beta_1, \gamma_1)$, for $\gamma_2 + \triangle < \gamma_1$. Since $(\alpha_1, \beta_2, \gamma_1) \succeq (\alpha_2, \beta_1, \gamma_1)$, for any outcome $\gamma_1 \in R$, by continuity there should exist a $\gamma_2 + \triangle < \gamma_1$ (very close to γ_1), such that $(\alpha_1, \beta_2, \gamma_2 + \triangle) \succeq (\alpha_2, \beta_1, \gamma_1)$ holds. As a result of increasing γ_2, because of strong monotonicity and d-transitivity, preference in (*i*) is not affected, i.e., by strong monotonicity $(\alpha_1', \beta_1, \gamma_1) \succeq (\alpha_2', \beta_2, \gamma_1) \succ (\alpha_2', \beta_2, \gamma_2 + \triangle)$, which implies $(\alpha_1', \beta_1, \gamma_1) \succ (\alpha_2', \beta_2, \gamma_2 + \triangle)$ by d-transitivity. So now rewriting (*i*), (*ii*), (*iii*), and (*iv*) we get (*i*) $(\alpha_1', \beta_1, \gamma_1) \succ (\alpha_2', \beta_2, \gamma_2 + \triangle)$, (*ii*) $(\alpha_1, \beta_2, \gamma_2 + \triangle) \succeq (\alpha_2, \beta_1, \gamma_1)$, (*iii*) $(\alpha_2, \beta_1', \gamma_1') \succeq (\alpha_1, \beta_2', \gamma_2')$, (*iv*) $(\alpha_2', \beta_2', \gamma_2') \succ (\alpha_1', \beta_1', \gamma_1')$. From (*i*) and (*ii*), we get $\alpha_1'\alpha_2' \succ_t \alpha_2\alpha_1$. From (*iii*) and (*iv*) we get $\alpha_2\alpha_1 \succ_t \alpha_1'\alpha_2'$. Such preferences violate preference trade-off consistency and the implication is proved for this case.

Case 3: There is a change in preference of (*iii*), but no change in (*ii*). We get (*i*) $(\alpha_1', \beta_1, \gamma_1) \succeq (\alpha_2', \beta_2, \gamma_2)$, (*ii*) $(\alpha_1, \beta_2, \gamma_2) \succeq (\alpha_2, \beta_1, \gamma_1)$, (*iii*) $(\alpha_2, \beta_1', \gamma_1') \preceq (\alpha_1, \beta_2', \gamma_2')$, (*iv*) $(\alpha_2', \beta_2', \gamma_2') \succ (\alpha_1', \beta_1', \gamma_1')$. Note that such preference is possible only if $\gamma_2' > \gamma_1'$. Otherwise if $\gamma_1' > \gamma_2'$, by strong monotonicity and preference in (*c*), we get $(\alpha_2, \beta_1', \gamma_1') \succ (\alpha_2, \beta_1', \gamma_2') \succeq (\alpha_1, \beta_2', \gamma_2')$, and by d-transitivity we get $(\alpha_2, \beta_1', \gamma_1') \succ (\alpha_1, \beta_2', \gamma_2')$, which contradicts the preference in (*iii*). So γ_2' should be greater than γ_1'.

Now we increase γ_1' to $\gamma_1' + \triangle$ such that preference in (*iii*) is reversed, i.e., $(\alpha_2, \beta_1', \gamma_1' + \triangle) \succeq (\alpha_1, \beta_2', \gamma_2')$ for $\gamma_1' + \triangle < \gamma_2'$. Since $(\alpha_2, \beta_1', \gamma_1') \succeq (\alpha_1, \beta_2', \gamma_1')$, for any outcome $\gamma_1' \in R$, by continuity there should exist a $\gamma_1' + \triangle < \gamma_2'$ (very close to γ_2'), such that $(\alpha_2, \beta_1', \gamma_1' + \triangle) \succeq (\alpha_1, \beta_2', \gamma_2')$ holds. As a result of increasing γ_1', because of strong monotonicity and d-transitivity, preference in (*iv*) is not affected, i.e., by strong monotonicity $(\alpha_2', \beta_2', \gamma_2') \succ (\alpha_2', \beta_2', \gamma_1' + \triangle) \succeq (\alpha_1', \beta_1', \gamma_1' + \triangle)$, which implies $(\alpha_2', \beta_2', \gamma_2') \succ (\alpha_1', \beta_1', \gamma_1' + \triangle)$ by d-transitivity. So now rewriting (*i*), (*ii*), (*iii*), and (*iv*) we get (*i*) $(\alpha_1', \beta_1, \gamma_1) \succeq (\alpha_2', \beta_2, \gamma_2)$, (*ii*) $(\alpha_1, \beta_2, \gamma_2) \succeq (\alpha_2, \beta_1, \gamma_1)$, (*iii*) $(\alpha_2, \beta_1', \gamma_1' + \triangle) \succeq (\alpha_1, \beta_2', \gamma_2')$, (*iv*) $(\alpha_2', \beta_2', \gamma_2') \succ (\alpha_1', \beta_1', \gamma_1' + \triangle)$. From (*i*) and (*ii*), we get $\alpha_1'\alpha_2' \succ_t \alpha_2\alpha_1$. From (*iii*) and (*iv*) we get $\alpha_2\alpha_1 \succ_t \alpha_1'\alpha_2'$. Such preferences violate preference trade-off consistency and the implication is proved for this case.

Case 4: There is a change in preference of (*ii*) and (*iii*). We get (*i*) $(\alpha_1', \beta_1, \gamma_1) \succeq (\alpha_2', \beta_2, \gamma_2)$, (*ii*) $(\alpha_1, \beta_2, \gamma_2) \preceq (\alpha_2, \beta_1, \gamma_1)$, (*iii*) $(\alpha_2, \beta_1', \gamma_1') \preceq (\alpha_1, \beta_2', \gamma_2')$, (*iv*) $(\alpha_2', \beta_2', \gamma_2') \succ (\alpha_1', \beta_1', \gamma_1')$.

Now applying steps in Case 3 and 4, we will get (*i*) $(\alpha_1', \beta_1, \gamma_1) \succ (\alpha_2', \beta_2, \gamma_2 + \triangle)$, (*ii*) $(\alpha_1, \beta_2, \gamma_2 + \triangle) \succeq (\alpha_2, \beta_1, \gamma_1)$, (*iii*) $(\alpha_2, \beta_1', \gamma_1' + \triangle) \succeq (\alpha_1, \beta_2', \gamma_2')$, (*iv*) $(\alpha_2', \beta_2', \gamma_2') \succ (\alpha_1', \beta_1', \gamma_1' + \triangle)$. From (*i*) and (*ii*), we get $\alpha_1'\alpha_2' \succ_t \alpha_2\alpha_1$. From (*iii*) and (*iv*) we get $\alpha_2\alpha_1 \succ_t \alpha_1'\alpha_2'$. Such preferences violate preference trade-off consistency and the implication is proved for this case.

Similarly, we can also prove for other cases of preferences among three-outcome prospects without a common gauge, if Fishburn independence is violated, then preference trade-off consistency will also be violated under the assumptions of Lemma 4.

Step 4: Now we have proved the lemma for up to three-outcome prospects. In this step we will prove it for prospects with more than three outcomes.

Consider four-outcome prospects with states s_1, s_2, s_3, and s_4. Note that for four-outcome prospects that satisfy F_4 and share a common outcome across a particular state, from Step 3 and preference trade-off consistency, the preferences satisfy Fishburn independence. That is preferences like 1) (a) $(\alpha_1', \beta_1, \gamma_1, \delta_1) \succeq (\alpha_2', \beta_2, \gamma_2, \delta_1)$, (b) $(\alpha_1, \beta_2, \gamma_1', \delta_1) \succeq (\alpha_2, \beta_1, \gamma_2', \delta_1)$, (c) $(\alpha_2, \beta_1', \gamma_2, \delta_1) \succeq (\alpha_1, \beta_2', \gamma_1, \delta_1)$, and (d) $(\alpha_2', \beta_2', \gamma_2', \delta_1) \not\succ (\alpha_1', \beta_1', \gamma_1', \delta_1)$ (same outcome δ_1 under s_4 across all prospects) and 2) (a) $(\alpha_1', \beta_1, \gamma_1, \delta_1) \succeq (\alpha_2', \beta_2, \gamma_1, \delta_2)$, (b) $(\alpha_1, \beta_2, \gamma_1, \delta_1') \succeq (\alpha_2, \beta_1, \gamma_1, \delta_2')$, (c) $(\alpha_2, \beta_1', \gamma_1, \delta_2) \succeq (\alpha_1, \beta_2', \gamma_1, \delta_1)$, and (d) $(\alpha_2', \beta_2', \gamma_1, \delta_2') \not\succ (\alpha_1', \beta_1', \gamma_1, \delta_1')$ (same outcome γ_1 under s_3 across all prospects), hold for all outcomes $\alpha_i, \alpha_i', \beta_i, \beta_i', \gamma_i, \gamma_i', \delta_i, \delta_i' \in \mathbb{R}^n$ and $i = 1, 2$.

Now we have to prove that Lemma 4 holds for four-outcome prospects with different outcomes across all states. For instance, we have to show that preferences like (i) $(\alpha_1', \beta_1, \gamma_1, \delta_1) \succeq (\alpha_2', \beta_2, \gamma_2, \delta_2)$, (ii) $(\alpha_1, \beta_2, \gamma_1', \delta_1') \succeq (\alpha_2, \beta_1, \gamma_2', \delta_2')$, (iii) $(\alpha_2, \beta_1', \gamma_2, \delta_2) \succeq (\alpha_1, \beta_2', \gamma_1, \delta_1)$, and (iv) $(\alpha_2', \beta_2', \gamma_2', \delta_2') \succ (\alpha_1', \beta_1', \gamma_1', \delta_1')$, that violate Fishburn independence is not possible. Note that prospects in (i), (ii), (iii) and (iv) do not have a common gauge, so the preferences do not violate preference trade-off consistency directly. So we follow the procedure in step 3, to show that (i), (ii), (iii), and (iv) violate preference trade-off consistency. As a first step, we interchange the outcome γ_1' and γ_2' in (ii) with γ_2 and γ_1 in (iii). Because of this change, the preferences in (ii) and (iii) could be reversed. For instance we consider preference (ii) is reversed as in case 2 of step 3 (if preference (iii) is also reversed we can follow the procedure in case 4 of step 3), we get (i) $(\alpha_1', \beta_1, \gamma_1, \delta_1) \succeq (\alpha_2', \beta_2, \gamma_2, \delta_2)$, (ii) $(\alpha_1, \beta_2, \gamma_2, \delta_1') \preceq (\alpha_2, \beta_1, \gamma_1, \delta_2')$, (iii) $(\alpha_2, \beta_1', \gamma_1', \delta_2) \succeq (\alpha_1, \beta_2', \gamma_2', \delta_1)$, and (iv) $(\alpha_2', \beta_2', \gamma_2', \delta_2') \succ (\alpha_1', \beta_1', \gamma_1', \delta_1')$. From (ii) $(\alpha_1, \beta_2, \gamma_2, \delta_1') \preceq (\alpha_2, \beta_1, \gamma_1, \delta_2')$, $\gamma_1 > \gamma_2$, otherwise if $\gamma_2 > \gamma_1$, we may get $(\alpha_1, \beta_2, \gamma_2, \delta_1') \succ (\alpha_1, \beta_2, \gamma_1, \delta_1') \succeq (\alpha_2, \beta_1, \gamma_1, \delta_2')$ and by d-transitivity, we will get $(\alpha_1, \beta_2, \gamma_2, \delta_1') \succ (\alpha_2, \beta_1, \gamma_1, \delta_2')$, which contradicts (ii). So $\gamma_1 > \gamma_2$. Now we can increase γ_2 so close to γ_1, such that preference in *ii* is reversed. Continuity allows us to find $\gamma_2 + \triangle$, so close to γ_1 such that $(\alpha_1, \beta_2, \gamma_2 + \triangle, \delta_1') \succeq (\alpha_2, \beta_1, \gamma_1, \delta_2')$. Preference in (i) $(\alpha_1', \beta_1, \gamma_1, \delta_1) \succeq (\alpha_2', \beta_2, \gamma_2 + \triangle, \delta_2)$ holds (by d-transitivity and strong monotonicity). Now we get a set of preferences (i) $(\alpha_1', \beta_1, \gamma_1, \delta_1) \succeq (\alpha_2', \beta_2, \gamma_2 + \triangle, \delta_2)$, (ii) $(\alpha_1, \beta_2, \gamma_2 + \triangle, \delta_1') \succeq (\alpha_2, \beta_1, \gamma_1, \delta_2')$, (iii) $(\alpha_2, \beta_1', \gamma_1', \delta_2) \succeq (\alpha_1, \beta_2', \gamma_2', \delta_1)$, and (iv) $(\alpha_2', \beta_2', \gamma_2', \delta_2') \succ (\alpha_1', \beta_1', \gamma_1', \delta_1')$. Now we interchange δ_1 and δ_2 in (iii) with δ_2' and δ_1' in (ii), and we follow a similar procedure as above, we finally get (i) $(\alpha_1', \beta_1, \gamma_1, \delta_1) \succeq (\alpha_2', \beta_2, \gamma_2 + \triangle, \delta_2 + \triangle')$, (ii) $(\alpha_1, \beta_2, \gamma_2 + \triangle, \delta_2 + \triangle') \succeq (\alpha_2, \beta_1, \gamma_1, \delta_1)$, (iii) $(\alpha_2, \beta_1', \gamma_1', \delta_1') \succeq (\alpha_1, \beta_2', \gamma_2', \delta_2')$, and (iv) $(\alpha_1', \beta_2', \gamma_2', \delta_2') \succ (\alpha_2', \beta_1', \gamma_1', \delta_1')$. Now from (i) and (ii), we get $\alpha_1'\alpha_2' \succeq_t \alpha_2\alpha_1$ and from (iii) and (iv) we get $\alpha_2\alpha_1 \succ_t \alpha_1'\alpha_2'$, which violates preference trade-off consistency. Hence Lemma 4 is proved. We can show, similarly, that preference trade-off consistency is violated for all other four-outcome prospects that satisfies F_4, but violates Fishburn independence.

To prove Lemma 4 for four-outcome prospects, the key result we used from the step 3 was that the preference trade-off consistency holds among four-outcome prospects with one common outcome. To prove for prospects with $n > 4$ outcomes, we similarly assume preference trade-off consistency holds among n-outcome prospects with one common outcome (or $n - 1$ different outcomes). By mathematical induction we can extend the implication to n-outcome prospects. $\square$

Lemma 5. *If Assumption 1 holds, then Fishburn's (1990) representation holds with a strictly monotonic ψ_i function, i.e., for prospects $f = (f_1, \dots, f_n)$ and $g = (g_1, \dots, g_n)$, $f \succeq g$ if and only if $\sum \psi_i(f_i, g_i) \geq 0$, where ψ_i is continuous, strictly increasing (resp., decreasing) in the first (resp., second) argument, and skew-symmetric.*

Proof. Fishburn (1990) uses 4 axioms to derive a state-dependent regret representation. The axioms used by Fishburn (1990) are as follows: (a) Structure (connected and separated topological space), (b) Continuity, (c) Independence, and (d) Nonextremality. To prove Lemma 5, we have to show that Assumption 1 implies the Axioms (a) to (d) of Fishburn (1990).

Our continuity and strong monotonicity axiom implies continuity and nonextremality axiom of Fishburn (1990). Since our outcome set is an interval in $\mathbb{R}$, it is endowed with the structure that Fishburn (1990) requires. In Proposition 1, we have shown that, under Assumption 1, trade-off consistency implies preference trade-off consistency and preference trade-off consistency implies Fishburn independence. Thus since Assumption 1 implies Axiom (a) to (d) of Fishburn (1990), we get the Fishburn (1990) representation, i.e., for prospects $f = (f_1, \dots, f_n)$ and $g = (g_1, \dots, g_n)$, $f \succeq g$ if and only if $\sum \psi_i(f_i, g_i) \geq 0$, where ψ_i is continuous, and skew-symmetric. Since strong monotonicity holds, ψ_i is strictly increasing in the first argument and strictly decreasing in the second argument. $\square$

Assumption 2. *Assume Fishburn's (1990) representation with strictly monotonic ψ_i i.e., for prospects $f = (f_1, \dots, f_n)$ and $g = (g_1, \dots, g_n)$, $f \succeq g$ if and only if $\sum \psi_i(f_i, g_i) \geq 0$, where ψ_i is continuous, strictly increasing and skew-symmetric.*

Lemma 6. *If Assumption 1 and 2 hold, then $\forall i = 1 \dots n$ and $\forall j = 1, \dots, n$, $\psi_i = a_{i,j} \cdot \psi_j + c_{i,j}$, where $a_{i,j} > 0$ and $c_{i,j}$ are constants.*

Proof. Consider the following indifferences $\alpha_i f \sim \beta_i g$, $\beta_i f \sim \gamma_i g$, $\alpha_{i'} f' \sim \beta_{i'} g'$ for a state $s_i, i' \in 1, \dots, n$. In a small neighborhood of β such indifferences can be found. Then trade-off consistency implies $\beta_{i'} f' \sim \gamma_{i'} g'$. Applying the Fishburn (1990) representation, we get $\psi_i(\alpha, \beta) + \sum_{j \neq i} \psi_j(f_j, g_j) = 0$, $\psi_i(\beta, \gamma) + \sum_{j \neq i} \psi_j(f_j, g_j) = 0$, $\psi_{i'}(\alpha, \beta) + \sum_{j \neq i'} \psi_j(f'_{j'}, g'_{j'}) = 0$, $\psi_{i'}(\beta, \gamma) + \sum_{j \neq i'} \psi_j(f'_{j'}, g'_{j'}) = 0$. From the four equalities, we get

$$\psi_i(\alpha, \beta) - \psi_i(\beta, \gamma) = \psi_{i'}(\alpha, \beta) - \psi_{i'}(\beta, \gamma). \tag{D.1}$$

For Eq. (D.1) to be satisfied, ψ_i and $\psi_{i'}$ should be related by an affine transformation, i.e., $\psi_i = a \cdot \psi_{i'} + c$, where a and c are constants. Strong monotonicity requires $a > 0$.

Similarly if we can find indifferences $\alpha_{i'} f' \sim \beta_{i'} g'$, $\beta_{i'} f' \sim \gamma_{i'} g'$, $\alpha_j f'' \sim \beta_j g''$ in the small neighborhood of β, then trade-off consistency implies $\beta_j f'' \sim \gamma_j g''$. Now applying the Fishburn (1990) representation, we get $\psi_{i'}(\alpha, \beta) + \sum_{j \neq i'} \psi_j(f'_j, g'_j) = 0$, $\psi_{i'}(\beta, \gamma) + \sum_{j \neq i'} \psi_j(f'_j, g'_j) = 0$, $\psi_j(\alpha, \beta) + \sum_{i \neq j} \psi_i(f''_i, g''_i) = 0$, $\psi_j(\beta, \gamma) + \sum_{i \neq j} \psi_{i'}(f''_i, g''_i) = 0$. From the four equalities, we get

$$\psi_{i'}(\alpha, \beta) - \psi_{i'}(\beta, \gamma) = \psi_j(\alpha, \beta) - \psi_j(\beta, \gamma). \tag{D.2}$$

For Eq. (D.2) to be satisfied, ψ_j and $\psi_{i'}$ should be related by an affine transformation, i.e., $\psi_{i'} = a' \cdot \psi_j + c'$, where a' and c' are constants. Strong monotonicity again requires $a' > 0$. Now since we know that $\psi_i = a \cdot \psi_{i'} + c$ we get $\psi_i = a \cdot a' \cdot \psi_j + ac' + c = a'' \psi_j + c''$, where $a'' = a \cdot a' > 0$ and $c'' = ac' + c$. Thus we have shown that any ψ_i and ψ_j, $j \neq i$ are related by an affine function. By a similar reasoning we can show that all ψ_i are related by affine functions.

The same proof above could be adapted for a general state space S and different partitions s_i and $s_{i'}$. □

Lemma 7. *If Assumption 1 and 2 hold, then for prospects $f = (f_1, \dots, f_n)$ and $g = (g_1, \dots, g_n)$, $f \succeq g$ if and only if $\sum p_i \cdot \psi(f_i, g_i) \geq 0$, where ψ is continuous, strictly increasing and skew-symmetric, $p_i \geq 0$ and $\sum_{i=1}^n p_i = 1$.*

Proof. From Lemma 6, we know that $\psi_i = a_j \cdot \psi_j + c_j$, $a_j > 0$ and c_j are constants, $\forall j = 1, \dots, n$. Now let $\psi = \psi_i$ and $p_j = a_j / (\sum_{k=1}^n a_k) \forall j$. Note that $a_k = 1$ for $k = 1$, so that the denominator $\sum_{k=1}^n a_k$ is positive. Since ψ_i is continuous, strictly increasing (resp., strictly decreasing) in the first (resp., second) argument, and skew-symmetric, ψ is also continuous, strictly increasing (resp., strictly decreasing) in the first (resp., second) argument, and skew-symmetric. Thus $f \succeq g \longleftrightarrow \sum \psi_i(f_i, g_i) \geq 0 \longleftrightarrow \sum p_i \psi(f_i, g_i) \geq 0$. Thus we have derived the regret theory representation in Eq. (2.1) with a continuous ψ and unique subjective probabilities p_i. □

Lemma 8. *If Assumption 1 and 2 hold, then $f = (f_1, \dots, f_n)$ and $g = (g_1, \dots, g_n)$, $f \succeq g$ if and only if $\sum_{i=1}^n p_i \cdot \psi(f_i, g_i) \geq 0 \longleftrightarrow \sum_{i=1}^n p_i Q(u(f_i) - u(g_i)) \geq 0$.*

Proof. We take a small outcome $\alpha_0 = \beta_0$ and a somewhat larger outcome α_1 (very close to α_0) and continuity allows us to find $\beta_1 > \beta_0$ such that $(\alpha_1, \beta_0, f_3, \dots, f_n) \sim (\alpha_0, \beta_1, g_3, \dots, g_n)$ holds. For small enough $\beta_0 (= \alpha_0)$ and α_1 such a β_1 exists. For as many i as possible, we define α_{i+1} by $(\alpha_{i+1}, \beta_0, f_3, \dots, f_n) \sim (\alpha_i, \beta_1, g_3, \dots, g_n)$. Now by trade-off consistency, we get $\alpha_{i+1} \ominus \alpha_i \sim_t \alpha_i \ominus \alpha_{i-1} \sim_t \alpha_{i-1} \ominus \alpha_{i-2} \sim_t \dots\dots \sim_t \alpha_1 \ominus \alpha_0$. This also means from the representation derived in Lemma 7, $\psi(\alpha_{k+1}, \alpha_k) = \dots = \psi(\alpha_1, \alpha_0) = b$, where $b = \frac{p_2}{p_1} \cdot \psi(\beta_1, \beta_0) + \frac{1}{p_1} \sum_{i=3}^n p_i \cdot \psi(f_i, g_i)$. For as many j as possible, similarly, we define β_{j+1} by $(\alpha_1, \beta_j, f_3, \dots, f_n) \sim (\alpha_0, \beta_{j+1}, g_3, \dots, g_n)$, we get $\beta_{j+1} \ominus \beta_j \sim_t \beta_j \ominus \beta_{j-1} \sim_t \beta_{j-1} \ominus \beta_{j-2} \sim_t \dots\dots \sim_t \beta_1 \ominus \beta_0$. Again from representation derived in Lemma 7 this *infers* $\psi(\beta_{k+1}, \beta_k) = \dots = \psi(\beta_1, \beta_0) = b'$, where $b' = \frac{p_1}{p_2} \cdot \psi(\alpha_1, \alpha_0) + \frac{1}{p_2} \sum_{i=3}^n p_i \cdot \psi(f_i, g_i)$. Now by trade-off consistency, from $(\alpha_{i+1}, \beta_0, f_3, \dots, f_n) \sim (\alpha_i, \beta_1, g_3, \dots, g_n)$ and $(\alpha_1, \beta_j, f_3, \dots, f_n) \sim (\alpha_0, \beta_{j+1}, g_3, \dots, g_n)$, we also get

$$(\alpha_{i+1}, \beta_j, f_3, \dots, f_n) \sim (\alpha_i, \beta_{j+1}, g_3, \dots, g_n) \tag{D.3}$$

We will now find $\alpha_{0.5}$ such that $(\alpha_{0.5}, \beta_0, f_3, \dots, f_n) \sim (\alpha_0, \beta, g_3, \dots, g_n)$ and $(\alpha_1, \beta_0, f_3, \dots, f_n) \sim (\alpha_{0.5}, \beta, g_3, \dots, g_n)$ hold for some β. Representing the above indifferences in terms of ψ, we get (i) $\psi(\alpha_{0.5}, \alpha_0) = \frac{p_2}{p_1} \cdot \psi(\beta, \beta_0) + \frac{1}{p_1} \sum_{i=3}^n p_i \cdot \psi(f_i, g_i)$ and (ii) $\psi(\alpha_1, \alpha_{0.5}) = \frac{p_2}{p_1} \psi(\beta, \beta_0) + \frac{1}{p_1} \sum_{i=3}^n p_i \cdot \psi(f_i, g_i)$. We have to find $\alpha_{0.5}$ such that (i) and (ii) hold. To find such $\alpha_{0.5}$, we start with β close to β_0 and we find outcome f_1 such that $\psi(f_1, \alpha_0) = \psi(\beta, \beta_0) + \sum_{i=3}^n \psi(f_i, g_i)$ and substituting this f_1, we find f_1' such that $\psi(f_1', f_1) = \psi(\beta, \beta_0) + \sum_{i=3}^n \psi(f_i, g_i)$. We increase β until we reach $f_1' = \alpha_1$. Since for $\beta = \beta_1$, $f_1' = \alpha_2$ and for $\beta = \beta_0$, $f_1' = \alpha_0$, and since ψ is strictly increasing, by continuity of ψ there should be a β in the interval (β_0, β_1) such that $f_1' = \alpha_1$. That f_1 for which $f_1' = \alpha_1$ would be $\alpha_{0.5}$, i.e., $\alpha_{0.5} = f_1$ if $f_1' = \alpha_1$. Now since such a $\alpha_{0.5}$ exists, from the indifferences above, we get $\psi(\alpha_1, \alpha_{0.5}) = \psi(\alpha_{0.5}, \alpha_0) = b''$, where $b'' = \psi(\beta, \beta_0) + \sum_{i=3}^n \psi(f_i, g_i)$. Since $\alpha_1 > \alpha_{0.5} > \alpha_0$,

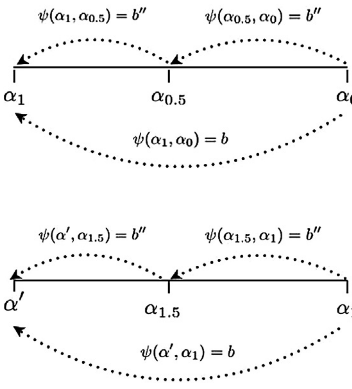

Fig. D.1. The ψ distance between outcomes.

we indicate $\psi(\alpha_1,\alpha_0)$ as a function F of $\psi(\alpha_1,\alpha_{0.5}) + \psi(\alpha_{0.5},\alpha_0)$. Let $\psi(\alpha_1,\alpha_0) = b = F(\psi(\alpha_1,\alpha_{0.5}) + \psi(\alpha_{0.5},\alpha_0)) = F(2b'')$.

Similarly by fixing β, we elicit $\alpha_{1.5}$ such that $(\alpha_{1.5},\beta_0, f_3,\dots, f_n) \sim (\alpha_1,\beta, g_3,\dots, g_n)$ and α' such that $(\alpha',\beta_0, f_3,\dots, f_n) \sim (\alpha_{1.5},\beta, g_3,\dots, g_n)$. Representing the above indifferences in terms of ψ, we get (i) $\psi(\alpha_{1.5},\alpha_1) = \frac{p_2}{p_1}\cdot\psi(\beta,\beta_0) + \frac{1}{p_1}\sum_{i=3}^n p_i \cdot \psi(f_i, g_i)$ and (ii) $\psi(\alpha',\alpha_{1.5}) = \frac{p_2}{p_1}\psi(\beta,\beta_0) + \frac{1}{p_1}\sum_{i=3}^n p_i \cdot \psi(f_i, g_i)$. Note that the right hand side of (i) and (ii) are identical and equivalent to b''. Therefore $\psi(\alpha_{1.5},\alpha_1) = \psi(\alpha',\alpha_{1.5}) = b''$. Again $\alpha' > \alpha_{1.5} > \alpha_1$ (see Fig. D.1), we get $\psi(\alpha',\alpha_1) = F(b'' + b'') = F(\psi(\alpha',\alpha_{1.5}) + \psi(\alpha_{1.5},\alpha)) = F(2b'') = b$. Since $\psi(\alpha_2,\alpha_1) = b$, and ψ is strictly increasing, we get $\alpha' = \alpha_2$. Now we know that $\psi(\alpha_2,\alpha_{1.5}) = \ldots = \psi(\alpha_{0.5},\alpha_0) = b''$ which implies, by trade-off consistency, $\alpha_2 \ominus \alpha_{1.5} \sim_t \alpha_{1.5} \ominus \alpha_1 \sim_t \ldots \sim_t \alpha_{0.5} \ominus \alpha_0$. By extending the argument we will get: $\alpha_{i+0.5} \ominus \alpha_i \sim_t \alpha_i \ominus \alpha_{i-0.5} \sim_t \alpha_{i-0.5} \ominus \alpha_{i-1} \sim_t \ldots.. \sim_t \alpha_{0.5} \ominus \alpha_0$ and thereby $\psi(\alpha_{i+0.5},\alpha_i) = \ldots = \psi(\alpha_{0.5},\alpha_0) = b''$. We can apply the same logic and show that for smaller differences in indices of standard sequences like 0.25, 0.125 and so on, only the difference between indices matters.

In other words, starting with standard sequence outcome $\alpha_{n'}$, we can show for any $i < n'$, that $\psi(\alpha_{k+i},\alpha_k) = \psi(\alpha_{j+i},\alpha_j)\forall k, j$. This means that the value of $\psi(\alpha_{k+i},\alpha_k)$ depends only on the difference between the indices of α_{k+i} and α_k. Therefore we can represent $\psi(\alpha_{k+i},\alpha_k)$ by $Q(u(\alpha_{k+i}) - u(\alpha_k))$, where the utility of outcomes are the affine transformation of their indices, i.e., $u(\alpha_{k+i}) = a\cdot(k+i)+c$ and $u(\alpha_k) = a\cdot k+c$, where c and $a > 0$, are constants. The function Q is such that $Q(u(\alpha_{k+i}) - u(\alpha_k)) = \psi(\alpha_{k+i},\alpha_k)$. As ψ is continuous and strictly increasing (resp., strictly decreasing) in the first (resp., second) argument, Q is also continuous and strictly increasing. The skew symmetry of ψ, implies the skew symmetry of Q: $Q(\alpha) = -Q(-\alpha)$.

By a similar argument as above, we can also show that $\psi(\beta_{k+i},\beta_k) = Q(u(\beta_{k+i}) - u(\beta_k))$. From Eq. (D.3) we know that $(\alpha_{i+1},\beta_j, f_3,\dots, f_n) \sim (\alpha_i,\beta_{j+1}, g_3,\dots, g_n)$, so the utility of difference between successive $\alpha's$ and $\beta's$ should be related by a constant a' i.e., $u(\beta_{k+1}) - u(\beta_k) = a'\cdot(u((\alpha_{k+1}) - u(\alpha_k))$ which implies $u(\beta_i) = a'(u(\alpha_i))$ as $\alpha_0 = \beta_0$. This constant a' can be estimated from $p_1\cdot Q(u(\alpha_{i+1}) - u(\alpha_i)) - p_2\cdot Q(a'\cdot(u(\alpha_{j+1}) - u(\alpha_i)) + \sum_{i=3}^n p_i\cdot Q(u(f_i) - u(g_i)) = 0$. Thus we have shown $\sum_{i=1}^n p_i \cdot \psi(f_i, g_i) \geq 0 \longleftrightarrow \sum_{i=1}^n p_i Q(u(f_i) - u(g_i)) \geq 0$. We have therefore derived u and Q function from the ψ function. Since ψ_i is unique up to unit in Fishburn (1990) representation, ψ is also unique upto unit i.e., ψ can be replaced by a function

$a \cdot \psi$ where $a > 0$, without affecting the preference. As $\psi(f_i, g_i) = Q(u(f_i) - u(g_i))$, Q is also unique up to unit. Since the regret theory representation depends only on the difference between the utilities of the outcomes, the function u is unique up to both unit and level i.e., it can be replaced by a function $u' = a \cdot u + c$ where $a > 0$ and any real c. □

Appendix E

We exploit the similarity between the axioms of EU and regret theory to illustrate the relationship between the two theories. We also illustrate the ramifications of dropping transitivity altogether, by discussing the relationship between regret theory and a theory that fully drops that axiom. To provide extra visual intuition, we detail the indifferences between prospects under EU, regret theory, and a completely intransitive theory (i.e., a theory that fully drops transitivity).

Relationship between regret theory and expected utility

Consider a prospect (α_1, α_2). The states s_1 and s_2 partition the state space S, such that $s_1 \cup s_2 = S$. We take a small outcome $\alpha_0 = \beta_0$ and a somewhat larger outcomes α_1. Then we define $\beta_1(> \beta_0)$ by

$$(\alpha_1, \beta_0) \sim (\alpha_0, \beta_1); \tag{E.1}$$

for as many i as possible, we define α_{i+1} by

$$(\alpha_{i+1}, \beta_0) \sim (\alpha_i, \beta_1); \tag{E.2}$$

likewise, for as many j as possible, we define β_{j+1} by

$$(\alpha_1, \beta_j) \sim (\alpha_0, \beta_{j+1}). \tag{E.3}$$

In Eq. (E.1) and Eq. (E.2) by defining β_1 and β_0 as gauge outcomes we get the standard sequence of outcomes α_i, α_{i+1} such that $\alpha_{i+1} \ominus \alpha_i \sim_t \alpha_1 \ominus \alpha_0$. Similarly, in Eq. (E.3) and Eq. (E.1), by defining α_1 and α_0 as gauge outcomes we get the standard sequence of outcomes $\beta_{j+1} \ominus \beta_j \sim_t \beta_1 \ominus \beta_0$. So for each i, we have $\alpha_{i+1} \ominus \alpha_i \sim_t \alpha_1 \ominus \alpha_0$ and Eq. (E.3), by trade-off consistency, implies

$$(\alpha_{i+1}, \beta_j) \sim (\alpha_i, \beta_{j+1}), \forall j. \tag{E.4}$$

That is, for a prospect, decreasing one subscript by 1 and increasing the other by 1 does not affect the preference value. Now we discuss the important role played by the transitivity of preference relation $\succeq$. Suppose transitivity of $\succeq$ holds, then $(\alpha_{i+1}, \beta_j) \sim (\alpha_i, \beta_{j+1})$ and $(\alpha_i, \beta_{j+1}) \sim (\alpha_{i-1}, \beta_{j+2}) \implies (\alpha_{i+1}, \beta_j) \sim (\alpha_{i-1}, \beta_{j+2})$. By repeated application of this logic, we show that increasing one subscript by any arbitrary number and decreasing the other subscript by the same arbitrary number does not affect the preference value, i.e., $(\alpha_{i+k}, \beta_j) \sim (\alpha_i, \beta_{j+k})$. Suppose the transitivity does not hold for all prospects (as in our behavioral foundation), then we cannot repeatedly apply this logic and we cannot infer whether (α_{i+k}, β_j) is *indifferent* to (α_i, β_{j+k}) for any arbitrary number k. This is where *regret theory begins to deviate from expected utility*. We illustrate it using Example 1.

Example 1. Consider the following assumptions for u and Q under regret theory: $Q(\alpha) = e^{\alpha} - 1$ for $\alpha \geq 0$ and $Q(\alpha) = 1 - e^{-\alpha}$ for $\alpha \leq 0$ (Q is exponential, skew-symmetric, continuous, and strictly increasing) and $u(\alpha_i) = 3 \cdot u(\beta_i) = 3i$. From trade-off consistency we know that Eq. (E.4) holds. So we get $\frac{p(s_1)}{p(s_2)} = \frac{Q(1)}{Q(3)}$, which gives $p(s_1) = \frac{e-1}{e^3+e-2}$ and $p(s_2) = \frac{e^3-1}{e^3+e-2}$. Under regret theory, the value of a prospect (α_{i+1}, β_j) with respect to prospect (α_i, β_{j+1}) is given by,

$$p(s_1)Q(u(\alpha_{i+1}) - u(\alpha_i)) + p(s_2)Q(u(\beta_{j+1}) - u(\beta_{j+2}) \\ = \frac{(e-1)(e^3-1)}{e^3+e-2} - \frac{(e^3-1)(e-1)}{e^3+e-2} = 0. \tag{E.5}$$

Eq. (E.5) implies $(\alpha_{i+1}, \beta_j) \sim (\alpha_i, \beta_{j+1}) \sim (\alpha_{i-1}, \beta_{j+2})$. Now calculating the regret theory value of the prospect (α_{i+1}, β_j) with respect to prospect $(\alpha_{i-1}, \beta_{j+2})$, we get

$$p(s_1)Q(u(\alpha_{i+1}) - u(\alpha_{i-1})) + p(s_2)Q(u(\beta_j) - u(\beta_{j+2}) \\ = \frac{(e-1)(e^6-1)}{e^3+e-2} - \frac{(e^3-1)(e^2-1)}{e^3+e-2} = 27.37 > 0. \tag{E.6}$$

Eq. (E.6) implies $(\alpha_{i+1}, \beta_j) \succ (\alpha_{i-1}, \beta_{j+2})$. From Eq. (E.5) and Eq. (E.6), we observe that $(\alpha_{i+1}, \beta_j) \sim (\alpha_i, \beta_{j+1})$, $(\alpha_i, \beta_{j+1}) \sim (\alpha_{i-1}, \beta_{j+2})$, but $(\alpha_{i+1}, \beta_j) \succ (\alpha_{i-1}, \beta_{j+2})$. Thus we obtain a *violation of transitivity under regret theory* for a *two-outcome prospect.*

However, when we consider a linear Q, i.e., $Q(\alpha) = b\alpha$, where b is a positive constant, we get $P(s_1) = \frac{1}{4}$ and $P(s_2) = \frac{3}{4}$ and the value of prospect (α_{i+k}, β_j) with respect to prospect (α_i, β_{j+k}) is zero i.e., $(\alpha_{i+k}, \beta_j) \sim (\alpha_i, \beta_{j+k})$, $\forall k$. Thus the example illustrates the role of a non-linear Q function in causing intransitive indifferences.

Indifferences under regret theory and expected utility

The indifferences under regret theory and expected utility are depicted in Figs. E.1 and E.2, respectively, together with a grid of outcomes constructed using the trade-off sequence. Each dot on the grid indicates a prospect that gives outcome α_i under state s_1 and outcome β_i under state s_2. We use two different types of lines, bold or dotted, to indicate the *indifference* between two prospects. For instance, the indifference between prospects (α_1, β_1) and (α_0, β_2), (α_0, β_1) and (α_1, β_0) in Fig. E.1 is indicated by a unique bold (dotted) line. However if two prospects are connected by both types, bold and dotted (for example prospects (α_2, β_0) and (α_0, β_2) in Fig. E.1), then the two prospects *need not be indifferent*. Note that the entire grid of outcomes is constructed by fixing the states s_1 and s_2. For different partitions we may get different grids.

The distinguishing feature of regret theory is that it allows for intransitivity of the preference relation $\succeq$. As a result, regret theory allows for different possible indifferences between the prospects in the grid. For example, in Fig. E.1, although the prospect (α_1, β_1) is indifferent to prospects (α_0, β_2) and (α_2, β_0), the prospects (α_0, β_2) and (α_2, β_0) *need not be* indifferent to each other, i.e., $(\alpha_0, \beta_2) \sim (\alpha_2, \beta_0)$ *need not* hold. Similarly, we do not know if other prospects such as (α_3, β_0) and (α_0, β_3), (α_3, β_1) and (α_1, β_3) are indifferent to each other. Regret theory allows for different indifferences between such prospects. In Fig. E.1, we show that two prospects of the form (α_{i+1}, β_j) and (α_i, β_{j+1}) that differ in the subscripts of standard sequence outcomes by one, are indifferent due to trade-off consistency (Eq. (E.4)). However the indifference need not hold between prospects that differ in subscript by a number other than one, i.e., prospects of the form (α_{i+k}, β_j) and (α_i, β_{j+k}) where $k \neq 1$. The absence of transitivity allows regret theory to have different indifferences between such prospects. This extra flexibility in the choice of indifferences is what differentiates regret theory from expected utility.

When imposing the transitivity axiom, we get the indifferences in Fig. E.2. Transitivity restricts the flexibility of the indifferences in Fig. E.1. For example in Fig. E.2, we observe that

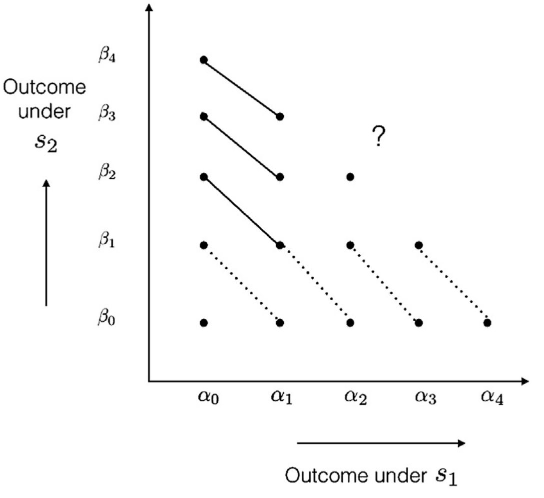

Fig. E.1. Indifferences under regret theory.

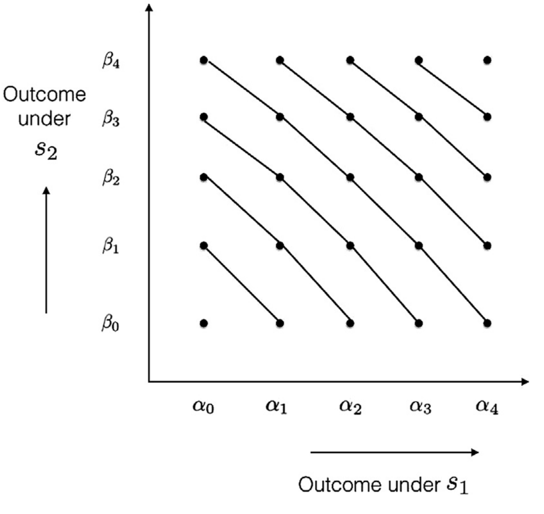

Fig. E.2. Indifference curves under expected utility (EU).

transitivity forces prospects (α_0, β_2), (α_2, β_0), and (α_1, β_1) to lie on the same indifference curve, which was not the case in Fig. E.1. Such indifference curves require Q to be linear and necessitate an expected utility representation.

The indifferences generated by regret theory with an exponential specification for the Q function (as in Example 1) are shown in Fig. E.3. In Fig. E.3, if two prospects are connected by one type of line (bold or dotted), only then the two prospects are *indifferent*. Otherwise, the two prospects are not indifferent. We observe that, in Fig. E.3, two prospects of the form (α_{i+1}, β_j) and (α_i, β_{j+1}) that differ in subscript by one are *indifferent*. However, the indifference does not hold between prospects that differ in subscript by a number other than one, i.e., $(\alpha_{i+k}, \beta_j) \nsim (\alpha_i, \beta_{j+k})$, for $k \neq 1$.

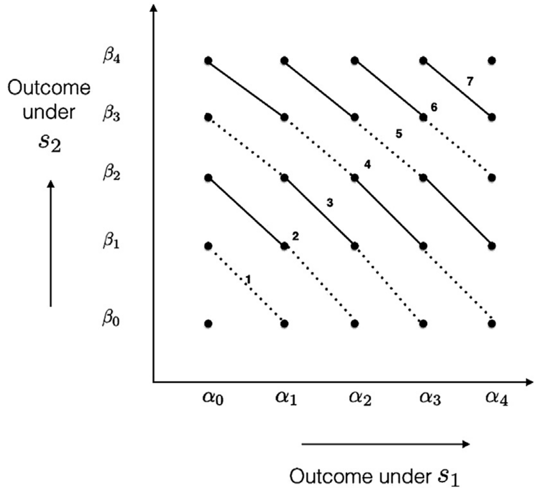

Fig. E.3. An example of indifference curves under regret theory.

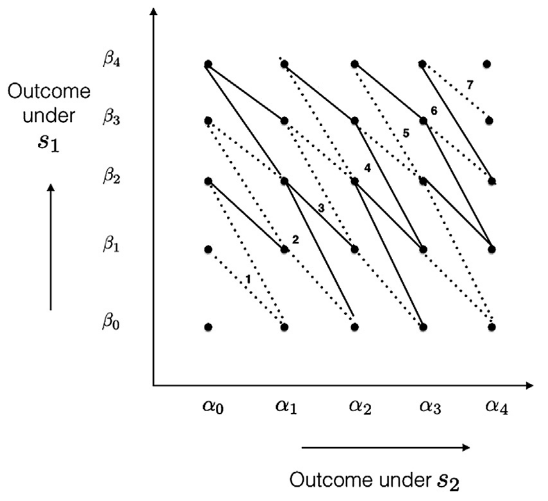

Fig. E.4. An example of indifference curves under a completely intransitive theory.

The role of d-transitivity

Regret theory retains a vestige of transitivity, i.e., the axiom of d-transitivity. To understand the role of d-transitivity, we compare the indifferences of regret theory with those of a completely intransitivity theory, i.e., a theory that gives up d-transitivity. Comparing the indifferences under regret theory with the indifferences of a theory that gives up d-transitivity, provides insights on the role of d-transitivity axiom. Fig. E.4 shows the indifferences of a completely intransitive theory i.e., without d-transitivity axiom.

Fig. E.4 shows that such a theory offers more flexibility in the shape of indifference curve compared to regret theory. For example, under regret theory, as d-transitivity axiom holds, the prospects (α_1, β_1) and (α_0, β_3), (α_0, β_4) and (α_1, β_2) *cannot be indifferent* to each other. However a completely intransitive theory allows indifferences between such prospects. The role of d-transitivity axiom is understood by comparing the indifference in the Figs. E.3 and E.4. In the Figs. E.3 and E.4, the prospects at different levels are indicated by $1, \ldots, 7$. Level refers to the prospects along the same diagonal. Prospects at a particular level have at least an outcome under one event greater than the outcomes of all the prospects at a lower level. Prospects at higher levels have better outcomes. The d-transitivity axiom separates prospects at different levels from one another i.e., prospects at different levels do not have any indifference curve between them. For example in Fig. E.1, there is no indifference curve between prospects (α_2, β_0), (α_1, β_1) and (α_0, β_2) in level 2 and prospects (α_1, β_0) and (α_0, β_1) in level 1. D-transitivity along with strong monotonicity ensures that prospects at higher levels are preferred to the prospect at lower levels. This leads to a regret theory representation with a strictly increasing Q function. In the absence of d-transitivity, since there are different possible indifferences, we may not able to derive a representation. We discuss an example to illustrate how regret theory satisfies dominance-transitivity axiom but violates full transitivity.

Comparing the behavioral foundation of regret theory with the behavioral foundation of rank-dependent utility (RDU) theories (Köbberling and Wakker, 2003), we observe that regret theory weakens the transitivity axiom of EU, while the RDU theories weaken the trade-off consistency axiom of EU. As a consequence, the indifferences under regret theory are different from the indifferences under RDU. Thus both regret theory and RDU are able to accommodate the descriptive violations of EU by relaxing different axioms. So when regret theory is transitive, its behavior is not consistent with RDU theories but only with EU.

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